COMMENTARY

Dynamics versus optimization in non-convex environmental economics problems with a single welfare function

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Economics has a well-defined notion of equilibrium. Unlike mechanics or thermodynamics, economics does not include explicit theories of dynamics describing how equilibria are reached or whether they are stable. However, even simple economics problems such as maximization of a welfare function might sometimes be interpreted as dynamics problems. Here we consider when dynamics is relevant to welfare optimization problems involving a single decision-maker, for example, a social decision-maker maximizing a social welfare function. We suggest that dynamics occurs in case a welfare maximum can only be known through a sequence of local computations. These local computations give rise to a dynamical system, and the welfare optimum is also equilibrium. On the contrary, if the welfare function is known, then dynamics is irrelevant and the maximum can be chosen directly. The importance of choosing the right metaphor for an economics problem is discussed.

A common problem in climate change economics is determining the optimal level of mitigation[1], maximizing economic benefits of mitigation for the entire world, defined as the difference between benefits and costs[2]. In the absence of uncertainty, this would correspond to some level of global warming. Such welfare optima are conceptually different from equilibria in economics. A well-known introductory economics textbook describes equilibrium as a 'state in which an economic entity is at rest or in which the forces operating on the entity are in balance so that there is no tendency for change'[3]. However, the notion of equilibrium in economics also derives from mathematical optima, though this is usually elided[4].

Generally, however, these notions are different. If T^* degrees of global warming is believed to be the welfare maximizing solution, then policymakers might consider how to arrange, for example, through price or quantity instruments, that T^* were reached by the economic system. Whatever T' were actually reached would be the result of equilibria in the relevant markets for emissions generating activities, and in that sense T' could be called the equilibrium level of warming. Even if T' and T^* were to coincide, the welfare optimum and policy equilibrium would only contingently be the same.

The parallels that equilibrium thinking in economics has with physics are not only semantic. Neoclassical economics was inspired by physics, with economic equilibrium discussed as a balance between gradients of utilities, in analogy with force balance in mechanics[5,6]. However, unlike in mechanics, economic theory did not include description of processes leading to equilibrium. Subsequently efforts were made in economics to revise the analogy to equilibrium thermodynamics, to distinguish microscopic from macroscopic variables[6], but the basic difficulty remained. Economics omits dynamics of how equilibria are attained; imagine equilibrium thermodynamics without a kinetic theory of gases. Moreover, this feature of economics has advantages, avoiding messy explanations that would have had to include psychology, sociology, etc. which are difficult to represent. However, economic equilibrium invokes the same concept of equilibrium as physical theory, but without the scaffolding of dynamics.

For example, a price on CO_2 emissions would raise the supply curve of CO_2 emitting activities (i.e. make their supply more expensive), thereby reducing the equilibrium quantity of emissions. However, for reaching an equilibrium either or both sides of the relevant economic exchanges must gather information about the shape of supply and demand and estimate the equilibrium, either in a single step or as an iterative process, with the latter giving rise to a dynamical system. Economics does not make such an iterative process explicit.

A unique intersection between demand and supply curves occurs in case the functions describing the utility of consumption and the cost of production are convex. This guarantees that their derivatives, the demand and supply curves, are monotonic with unique intersection[3,7]. Such thinking underlying convex problems played a role in the marginalist approach to economics, involving comparison between marginal quantities (derivatives of costs, benefits, etc.), to reach an optimum[7,8].

In case of a single welfare function, the welfare optimizing solution can be treated as equating two marginal quantities, for example, that of a benefit and a cost. However, such balancing using algebra to identify the maximum does not turn such welfare maxima into equilibria, because equilibrium implies also that the agents involved have no tendency to change their behaviour to move them away from the equilibrium.

What then is the relation between these notions? We propose that a welfare maximum is also equilibrium when there is dynamics involved, for instance, when reached iteratively. Then a dynamics metaphor pertains to the problem, because the maximum is only reached iteratively, through repeated local computations; a sequence of iterative 'calculate–act–calculate–act–...' steps progressively leads to the maximum.

The 'calculate' step determines the subsequent 'act' step; for example, a firm might use estimates of marginal cost and marginal revenue in the neighbourhood of the present output to estimate a change in output in the next period that increases profit. Such calculation must be only approximate because these graphs are not known; otherwise, the maximum could be selected in a single step. Furthermore, each 'act' step creates conditions for a new 'calculate' step, and this information could not be learned in any other way. This is similar to a dynamical system taking small steps on a trajectory along a local gradient. This iterative process will lead to the optimal solution in a convex economics problem.

Applying the dynamics metaphor to economics problems implies that direct optimization is impossible. If the welfare function were already known, the maximum could be arrived at directly instead of relying on a slow iterative process.

The dynamics metaphor treats the economics actor as a gradient-following particle, with only local knowledge in a small neighbourhood of any choice made. This is a stronger assumption than utility maximization, entailing not only consistent preferences but also limited information that precludes making (globally) optimizing decisions in a single step. Implications are sharpest in non-convex economics problems where the two metaphors, dynamics and optimization, can yield different results.

With non-convexity there can be multiple stationary points of the welfare function[9], corresponding to multiple intersections between marginal curves being compared. Multiple local maxima can create difficulties for analysis. Pigou[10] in an early discussion of a non-convex situation in economics, writes 'in any industry where the conditions are such that there is not only one, but more than one, volume of investment which would yield a marginal social net product equal in value to that obtainable elsewhere, unless it so happens that the volume actually hit upon is that one of these which is the most favourable to the national dividend, an opening for improvement must exist. Benefit could be secured by a *temporary* bounty (or temporary protection) so arranged as to jerk the industrial system out of its present poise at a position of relative maximum, and induce it to settle down again at the position of absolute maximum'[10].

Pigou's metaphor is of dynamics with multiple stable equilibria, where the decision-maker for the firm (think of the CEO or plant manager) cannot choose the absolute maximum directly, because that is not known, as information is only local. The firm may have to be nudged using policy so that it can eventually progress from its local maximum to attain the absolute maximum. Such problems are objects for a dynamics metaphor. The corresponding optima are also equilibria, because they are reached through an approximate dynamical process and only persist if stable. Furthermore, they are objects for policy if either the system could settle into a local maximum that is worse than the global maximum, or if policymakers have a different view of social welfare, for example, because they have a longer time-horizon.

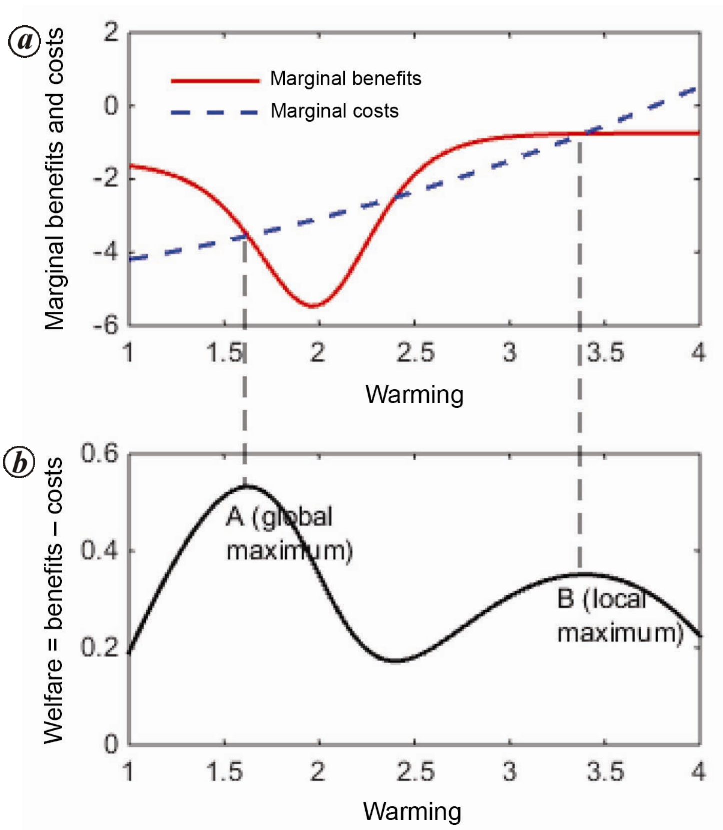

Figure 1. Illustration of a hypothetical non-convex climate change economics problem[11]. ***a***, Marginal benefits and costs as a function of warming. The marginal benefits function is not monotonic, since the benefits curve is non-convex. ***b***, Welfare function indicating global maximum *A* and local maximum *B*. If the welfare function is known, the global maximum *A* can be chosen directly.

How about choosing the optimal level of mitigation that maximizes a welfare function, the difference between benefits and costs of mitigation at various degrees of global warming? In this context, it has been pointed out[11] that impacts for some sectors of the economy might be non-convex. Therefore, contradictory to the conventional case of diminishing benefits to mitigation at lower levels of warming, there might be an intermediate temperature range where the benefits of mitigation are more sensitive to global warming. This can lead to multiple intersections of the marginal benefits and costs graphs, and hence multiple local maxima of the welfare function. Therefore, the authors raise the possibility of 'policy tipping' from a low-warming, high-cost, low-impact regime to a high-warming, low-cost, high-impact regime[11]. Figure 1 illustrates such a non-convex problem.

Here is an example where the entire welfare function is known, and the welfare optimum need not be attained by an iterative process. The best solution among the current possibilities can be chosen directly, and there is no dynamics involved (Figure 1 *b*)[12]. Therefore, there is no dynamical process ensuring stability of the chosen solution either, and the question of tendency for change cannot arise. In this problem, local optima of the welfare function are not economic equilibria despite corresponding to a balance between marginal costs and benefits of mitigation.

Returning to the general situation, we noted previously that an iterative process is relevant when only local information is available. In such cases the chosen welfare optima might be termed equilibria. This is despite the fact that economic theory generally does not make dynamics explicit, since the welfare optimum can only be reached by some unspecified dynamical process in such a situation.

The second condition for dynamics to apply is that each step in the process helps generate additional information for approaching the welfare optimum. From this, we deduce that the iterative process only applies where feedback from action to information occurs rapidly. Therefore, among such cases the relevant question is: is information only local? If the entire welfare function is known, then the correct metaphor is that of optimization, dynamics is not relevant, and the welfare optimum is not equilibrium. When faced with a non-convex economics problem, it is important to recognize the correct metaphor, which affects both the language employed to describe that problem as well as conclusions reached from analysis.

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12. As the authors furthermore discuss, failure to mitigate emissions rapidly could render what was previously the global optimum solution unreachable, due to physical and economic constraints. In reconsidering the policy goal the starting position would then be irreversibly different, presenting a new problem, and the original problem would then be improperly posed. This merely reflects the difficulty of simplifying a time-dependent dynamic optimization problem to the very different static one of choosing a single quantity, the maximum global warming.

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OPINION

Weedomics a need of time

Niraj Tripathi

Weeds are the plants, found simultaneously with crops and out-compete them in more or less every aspect. Competitive characters and tolerance to various abiotic and biotic stresses are the significant qualities which can be identified amongst a variety of weed species and can be transferred into crop plants for their advancement. Plant molecular biology includes the study of cellular processes, their genetic management and links with alterations in their adjoining. Advancement and accessibility of the sophisticated molecular tools offers us liberty to play with different metabolic pathways at molecular level and to transfer the desirable genetic materials into crop plants, thus breaking the reproductive barriers for interspecific and intergeneric transfer of the genetic material. Advanced plant molecular biology tools offer fabulous promises for elucidating these imperative traits from weed species in detail and further exploration for the diverse aspects of crop improvement in future. The large scale studies connecting entire genetic, structural, or functional machinery are called 'omics'. Major segments of omics consist of genomics, transcriptomics, proteomics, metallomics, metabolomics, ionomics and phenomics. At present these advances are frequently used in crop improvement. However, success of such approaches requires joint efforts from scientists to work mutually with expertise in weed science, molecular breeding and plant physiology. So the combined omic approaches in weed species (weedomics) for crop improvement may play a significant role in changing climatic conditions for food security (Figure 1).

Genomic helps in clarification of taxonomy and evolutionary relationships, uncover evidence of closely related species that cannot be morphologically distinguished (cryptic species) and hybridization events, elucidate methods of reproduction, determine population structure and origins of target weeds. These may help in biological control of invasive weeds. Advances in weed genomics can contribute to crop improvement by better understanding of the biological mechanisms and improved screening methods for selecting superior genotypes more efficiently which possess novel genes to provide resistance against adverse climatic conditions and biotic stresses. A wide range of defense mechanisms are activated that increases plant tolerance against adverse conditions to avoid damage imposed by stresses. The first step toward stress response is stress signal recognition and subsequent molecular, biochemical and physiological responses activated through signal transduction[1]. Understanding such responses is important for effective management of stress. Weeds possess higher level of stress tolerance and their transcriptome profiling may