## Measurement of coordinates of Nakśatras in Indian astronomy

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It is well known that ancient Indian calendar dwelled on the 27 naksiatra system for fixing the positions of the sun, moon and the planets. Several attempts to identify these 27 stars in the sky have yielded very precise results for stars bright enough not to be misidentified, which is not so for the fainter ones. The basis for identification is the coordinate system available in the texts. Here, we try to understand the ambiguity and offer a possible solution by using the measured coordinates, which have not been utilized for this purpose so far. This also provides clues on the techniques used for measuring the coordinates.

## The coordinate system

The foremost requirement for understanding the methods used in Indian astronomical texts is the coordinate system itself. Coordinates of stars in all texts of Indian astronomy are expressed in Dhruvaka and Vikśepa as explained in the figure; exact equivalent terms are not found in modern spherical astronomy texts. The great circle passing through the pole and the star, called the hour circle, intersects the ecliptic at a point B . The angle measured from the first point of Aries along the ecliptic to this point $B$ is called the Dhruvaka. The angle measured along the great circle passing through the pole of the ecliptic is called the Vikśepa.

Polar longitude and latitude are the terms that have been coined by later investigators. Modern text books define longitude and latitude relative the great circle passing through the pole of ecliptic; right ascension and declination are defined for the hour circle with reference to the equator.

All the sets of coordinates are interrelated by trigonometric relations to transform one set of coordinates to the other. Similarly, Dhruvaka and Vikśepa also can be converted to right ascension and declination. One of the earliest attempts is by Burgess ${ }^{1}$; subsequently others have come up with alternate formulae. A comparison of the different methods discusses the advantages of the Dhruvaka-Vikśepa system ${ }^{2}$.

The coordinates have been described as verses or phrases in various texts from Surya Siddhántha and its commentaries by various authors up to the 20th century - the last in the series by astronomer Chandrashekhar Samantha of Odisha.

Narahari Achar ${ }^{3}$ discussed the epochs as calculated using the names of the stars that are mentioned in texts in the context
of using Krittika for the sacrificial fire; he writes
'.. For, according to Pingree ${ }^{4}$, parts of nakSatras, Hasta, VisAkha, and perhaps SravaNa would also be on the equator on this date and this would contradict SB's claim that only kRttikAs "never swerve from the east." Further, he shows that there are about a dozen stars close to the equator. Of these, three are 30 minutes or less away from the equator, and four more are less than a degree away. There are additional four stars at about 1.5 degrees and the last one is about 2 degrees away from it.

Using the scheme of Pingree to identify the stars Hasta, visAkha, and SravaNa correspond to delta-Corvi, iota-Librae, and alpha-Aquilae respectively for 2927 BC , he further shows that none of the faint stars can be identified with the junction stars
or the yogatArAs themselves. It is true however, that there are stars which may be considered to be other members of the groups associated with some junction stars, i.e., asterisms. For Hasta, it is beta-Corvi, with a declination 1 degree 5 minutes, and epsilon-Corvi, with a declination of just 41 minutes. For visAkha, it may be sigma-Librae with a declination of mere 23 minutes; and/or delta-Scorpi with a declination of 57 minutes, and perhaps uttara proSThapada (Uttarabhadra), epsilon-Pegasi, with a declination of -1 degree 23 minutes
However, this objection does not really have any efficacy when one examines carefully the context under which that statement is made, namely, choosing the most auspicious nakSatra for performing agnyAdhAna. If the ritual of agnyAdhAna is to be done under kRttikAs because, "they never swerve from the east", then,


Figure 1. Coordinates Dhruvaka-Vikśepa and Right Ascension - Declination.

Pingree's point would be equivalent to stating that the same ritual could be performed equally well under Hasta, visAkha, and even uttara proSThapada. There may be other reasons why kRttikAs are preferred, such as the presiding deity being Agni. Thus the phrase "never swerve from east" cannot mean anything other than "rising heliacally exactly at the East Point", for, SB (Shathapatha Brahmana) itself declares: "udyanti pura etA[H]" "they rise in the east." On this point, sAyaNa also says in his exegesis "suddhaprAcyam evodyanti" "they rise in the true east."...'

The ambiguity in the identification of stars arises because the east west coordinates are influenced by the shift of the reference point, the First Point of Aries, owing to precession. The formulae for driving the north-south coordinate like declination involve this measurement and hence likely to carry forward the associated error. Zero point determination in Indian astronomical texts has been long debated. Saha and Lahiri ${ }^{5}$ reported in the Indian Calendar Reform Committee that the longitudes are available mainly for three epochs AD 340, 500 and 560 - Autumnal Equinox on $\alpha$ Vir and Vernal Equinox on $\zeta$ Psc. The inability to match the longitude values in subsequent texts was attributed to poor observational capabilities ${ }^{4}$.

We examine the identification of the stars named Yogatáras (junction stars) with the ones that are conventionally known to us now. One of the earliest lists is by Colebrook ${ }^{6}$ for the stars from the Surya Siddhánta. Subsequently various versions and commentaries have been made available.

Here is a comparison for the star Asvini from different texts - Surya Siddhánata, Mahendra Suri (who is believed to have translated the manual Yantra Rája of using the astrolabe into Sanskrit for the first time in 13th century, Pingree ${ }^{7}$ ), Malayendu ${ }^{8}$ (commentary on Yantra Rája in 15th century), Nityananda (who wrote Siddhántha Rája ${ }^{9}$ in 15th century), Padmanabha (author of Yantra Kirañávali in 16th century, Ohashi ${ }^{10}$ ), Putumana Somayaji (Karanapaddhati; between 16 and 18 century, Pai et al. ${ }^{11}$ ) and Siddhántha Darpaņa of Chandrashekhara Samanta. The choice of the texts is dictated by the fact they were all associated
with observational techniques. There are two dates mentioned in Samantha's text corresponding to 1869 and 1892 AD as the year of completion of the book.
Karanapaddhati gives longitudes and latitudes and are not included in Table 1. It provides methods to derive one set of coordinates from the other.

Table 1 gives a comparison of the values for different epochs; it may be recalled that the values given by Mahendra Suri are the same values provided by Ptolemy and corrected for precession (Ohashi ${ }^{10}$ ). The values from Nityananda, Padmanabha and Malayendu are tabulated in the context of the use of astrolabe, a measuring device borrowed from Arabs. As they are direct measurements, comparison becomes reliable. Malayendu lists another quantity named Paramónnatámśa which is a measured parameter and explains how to get the declination from this reading given that the latitude of the place is $28^{\circ} 38^{\prime}$. It is the maximum altitude, obviously corresponding to the meridian passage. Nityananda has not specified the latitude. Padmanabha mentions the year 1345 saka, which corresponds to 1423 AD . There is also a mention of a correction of $15^{\circ}$ for Dhruvaka.

Other texts like Grahalághava also list the coordinates which tally with the Surya Siddhánta values.

It is interesting to note that Malayendu provides both the declination and maximum altitude; the declination is given accurate to degrees minutes and seconds. There is a small difference in the values of declination derived directly from maximum altitude; we have provided this as correction $\Delta \delta$ (for the measured value) in Table 2. It can be either treated as instrumental error or a correction deliberately applied (maybe for refraction). Noticing that all the values of Dhruvakas (for 28 stars) end with $33^{\prime} 52^{\prime \prime}$ we can infer that they are calculated and/or corrected. Thus the measured quantities are only vikśepa and Paramónnatámśa (values terminate with degrees and minutes). There is a foot note for the table for Malayendu indicating that the Dhruvakas are for the period ranging from 1437 to 1637 AD . It is probably added in 1637 AD by a person who copied the text.
Karanapaddhathi and many other texts provide formulae for calculating declination from the longitude and latitude measures. The word Vikśepa, in these texts, refers to latitude itself.

The mode of observation for deriving the coordinates has been discussed by Saha and Lahiri ${ }^{5}$ by an armillary sphere with an ecliptic circle passing through stars like Pushya ( $\delta$ Cnc). The meridian transit was used for measuring the Dhruvaka and Vikśepa. As most of the records provided are integers, the accuracy is about 0.5 deg. Chandra Hari ${ }^{12}$ demonstrates the effectiveness of the procedure.
The bright stars which have been identified without any ambiguity can be used to verify the observations. The errors can be considered instrumental (Table 2).
It is interesting to note that the difference $\Delta \delta$ is a function of the declination. In other words, it is directly proportional to $a$, the altitude, clearly hinting at the error due to refraction; it is known that the correction is proportional to

$$
\cos a=-\tan \phi \tan \delta
$$

where $a$ is the maximum altitude (paramónnatámśa). The graph of $\cos a$ versus $\Delta \delta$ demonstrates this very clearly, as it is now known that the refraction correction varies as $\cos a$ (Figure 2). Thus it clearly establishes the value as an observational record.

## The 27 stars - a brightness scale

Now we proceed with the identification for the ambiguous cases. Nityananda provides the brightness as a scale called pramáņa, which is equivalent of the magnitude scale used today. The first (termed prathama pramáņa or ádyamána) is the brightest; the second brightest is termed dwimithi, the third as trimithi; it mentions even a fourth one as chaturtha pramána. These scales are specifically described in the middle of the text after the description of stars of Leo. It states that brightest ones are first magnitude; intermediate ones are third and there are thousands of stars fainter than the 6th. This value of the magnitude helps us in the identification. For example, if there are two stars very close to each other, based on the brightness scale, the correct one can be identified.
There are a few cases where brightness is not specified. For example for Krittika, it is described as looking like an arrow and like a fire. We find the notation agnimána - fire like for two more cases of nebulosities. One is the faint group in

Table 1. Comparison of coordinates of Aśvini ( $\beta$ Ari) in different texts

|  | Surya Siddhántha (~500 AD) | Nityananda <br> (1639 AD) Epoch for stars (600 AD) | Mahendra Suri Siddhánta Rája <br> (1400 AD) | Malayendu Commentary on Siddhánta Rája (1428 AD) ${ }^{\#}$ | Padmanabha <br> Yantra <br> Kiraņávali (1423 AD) mentions a correction of $15^{\circ}$ for Dhruvaka | Padmanabha's values after correction for precession to 1550 AD | Samanta <br> Chandrashekhara <br> Siddhánta <br> Darpaņa <br> (~1880 AD) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dhruvaka | $8^{\circ} 0^{\prime}$ | $14^{\circ} 6^{\prime}$ | $25^{\circ} 27^{\prime}$ | $25^{\circ} 21^{\prime} 52^{\prime \prime}$ | $+8^{\circ} 23$ |  | $10^{\circ} 07^{\prime}$ |
| Vikśepa | $+10^{\circ} 0^{\prime}$ | $+7^{\circ} 50{ }^{\prime}$ | $+7^{\circ} 20^{\prime}$ | $+7^{\circ} 20^{\prime}$ | +10 ${ }^{\circ} 55^{\prime}$ |  | +8 ${ }^{\circ} 29^{\prime}$ |
| Right ascension* | $7{ }^{\circ} 46^{\prime}$ | $12^{\circ} 52^{\prime}$ | $23^{\circ} 35^{\prime}$ | $23^{\circ} 32^{\prime}$ | $6^{\circ} 57{ }^{\prime}$ | $21^{\circ} 48^{\prime}$ | $9^{\circ} 11^{\prime}$ |
| Declination* | $12^{\circ} 54^{\prime}$ | $13^{\circ} 31^{\prime}$ | $17^{\circ} 14^{\prime}$ | $17^{\circ} 32^{\prime}$ | $14^{\circ} 25^{\prime}$ | $18^{\circ} 46^{\prime}$ | $8^{\circ} 27^{\prime}$ |
| Right ascension** | $8^{\circ} 30^{\prime}$ | $9^{\circ} 54^{\prime}$ | $20^{\circ} 45^{\prime}$ | 20 ${ }^{\circ} 5^{\prime}$ |  | $22^{\circ} 30^{\prime}$ | $26^{\circ} 49^{\prime}$ |
| Declination** | $12^{\circ} 59^{\prime}$ | $13^{\circ} 32^{\prime}$ | $17^{\circ} 48^{\prime}$ | $17^{\circ} 57^{\prime}$ |  | $18^{\circ} 34^{\prime}$ | $20^{\circ} 12^{\prime}$ |

*Calculated from Dhruvaka and Vikśepa.
**From Stellarium for the epoch.
\#The foot note to the table indicates another year 1637 AD; extended from 1437 AD.

Table 2. The error $\Delta \delta$ as derived from two measurements provided by Malayendu - the declination derived from maximum altitude and from Dhruvaka and Vikśepa measures

|  | Paramónnatámśa and <br> derived declination | Declination (kranti) <br> derived from <br> Dhruvaka | $\Delta \delta$ diff <br> derived from <br> columns $2 \& 3$ | Declination (1428 AD) <br> from stellarium |
| :--- | :---: | :---: | :---: | :---: |
| Star | $78^{\circ} 02^{\prime} / 16^{\circ} 40^{\prime}$ | $16^{\circ} 40^{\prime} 37^{\prime \prime}$ | $17^{\circ} 56^{\prime}$ |  |
| Asvini/ $\beta$ Ari (used for fixing latitude) | $76^{\circ} 53^{\prime} / 15^{\circ} 25^{\prime}$ | $15^{\circ} 22^{\prime} 77^{\prime \prime}$ | $0^{\circ} 0^{\prime} 37^{\prime \prime}$ | $0^{\circ} 2^{\prime} 53^{\prime \prime}$ |



Figure 2. Variation of the difference in (") between the values of the declination from the tables of Malayandu (see Table 2).

Cancer, including M 44 an open cluster. The other is the region near Milky Way.

Karaņapaddhathi, Graha Lághava and other texts give the same values as pro-
vided in Surya Siddhántha. Although Padmanabha lists the same values, he adds that a correction of $15^{\circ}$ should be made and that brings the date to about 15 th century (Ohashi ${ }^{10}$ ).

Table 3 lists the coordinates of all stars mainly from Nityananda; a comparison is provided from the sources cited above along with the relevant coordinates derived from Stellarium.
Bharani has been identified as 41 Ari; however based on the values declination we can find that it was $\gamma^{1}$ Ari. The identifications by Malayendu and Nityananda are two different stars; hence there is bound to be confusion if one uses only the longitude for fixing the star.
The last column of the Table 3 gives our identification in bold along with the justifications; the deviations with other works, if any, are also provided. We note that in case of faint stars the identification is different by different observers.

Revathi is identified as $\zeta \mathrm{Psc}$; however the coordinates agree with $\varepsilon$ Psc. It is quite possible that the two as a group was called Revathi. The former is now known to have a high proper motion based on Hipparcos data, implying that
Table 3. identification of stars (indicated in bold) from the catalogue of Nityananda and comparison with other catalogues

| Name | Dhruvaka $0, "$ | Vikśepa ○," | Magnitude | $\begin{gathered} \mathrm{RA} \\ \text { (calc) } \end{gathered}$ | $\begin{gathered} \text { Dec } \\ \circ, \prime \prime \end{gathered}$ | $\begin{gathered} \text { RA } \\ (600) \end{gathered}$ | $\begin{gathered} \text { Dec } \\ (600) \\ \circ, \end{gathered}$ | Max altitude | Drikshuddha corrected values | Declination from max a 01 | Identification/remarks |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Asvini | $14^{\circ} 6^{\prime}$ | $7^{\circ} 50^{\prime}$ | 3 | Oh 52m | 1331 | 00h 40m | $13^{\circ} 26^{\prime}$ |  |  |  | $\beta$ Ari |
|  | 8 | 10 |  | Oh 30m | 1311 |  |  |  |  |  | Surya Siddhantha (SS) from S\&L |
|  | 823 | 1055 |  | 01h 28m | 2016 | 01h 27m | 1820 |  |  |  | $\beta$ Ari Padmanabha's values AD 1428 |
|  | 2527 | 7.2 |  |  |  | 01h 27m | 1656 |  |  |  | Pingree identifies $\gamma$ Ari - Mahendra Suri's values |
| Malayendu's | 25/21/52 | 7/20 | 2 |  | 16/40/37 |  |  | 78/2 | 0/22/22/18 | 16/41 | Name Suratyena matches with Sheratan |
| Bharani | $23^{\circ} 31$ | $5^{\circ} 45$ |  | 01h 26m | 1555 | 01h 30m | 1500 |  |  |  | $\varepsilon$ Eri |
|  | $20^{\circ}$ | 12 |  | 01h 13m | 20 | 01h 30m | 2046 |  |  |  | 41 Ari (Abhyankar) based on SS coordinates |
|  | 217 | 1216 |  | 02h 14m | 2615 | 02h 21m | 2508 |  |  |  | 41 Ari Padmanabha's values (AD 1428) |
| Krittika | 3824 | $3{ }^{\circ} 45$ | ? | 02h 23m | 1808 | 02h 26m | 1848 |  |  |  | $\eta$ Tau magnitude not specified; Sharognimanam - looks like arrow of fire no ambiguity |
| Rohini | 4910 | $5^{\circ} 15^{\prime} \mathrm{S}$ | 1 | 03h 22m | 152 | 03h 56m | 152 |  |  |  | $\alpha$ Tau no ambiguity |
|  | 31/43/52 | 510 S |  |  |  |  | 15/22/7 | 76/53 | 2/2/23/23 | 15/25 | Malayendu-corrected Dhruvaka 62/23/23 |
|  |  |  |  |  |  |  |  |  |  |  | Davara name matches with Al deberan |
| Mrigasira | $687^{\prime \prime} 3{ }^{\prime \prime}$ | 13 30S | 1 | 04h 20m | 810 | 04h 18m | 749 |  |  |  | $\lambda$ Ori with dec difference |
|  | 63 | 10 S |  | 04h 03m | 1125 |  |  |  |  |  | SS values |
|  | 622 | 1095 |  | 05h 03m | 1226 | 05h 08m | 928 |  |  |  | $\lambda$ Ori Padmanabha's values AD 1428 |
| Ardra | 7807 | -7 12 | 1 | 05h 08m | 1551 | 05h 16m | 1620 |  |  |  | $\gamma$ Gem |
|  | 9633 | -39 10 |  |  |  |  |  |  |  |  | Pingree - $\alpha$ CMi based on Mahendra Suri's value |
|  | 6720 | -9 |  | 04h 21m | 1313 |  |  |  |  |  | SS values |
|  | 6555 | -116 |  | 06h 09m | 1635 | 06h 09m | 1641 |  |  |  | Padmanabha's values AD 1428 |
| Punarvasu | 9230 | 6 | 2 | 06h 11m | 30 | 06h 18m | 30 10' |  |  |  | $\beta$ Gem no ambiguity |
|  |  |  |  |  |  |  |  |  |  |  | Malayendu gives $\alpha$ Gem |
| Pusya | 10630 | 1 | ? | 07h12m | 2355 | 07h 23m | 2214 |  |  |  | $\delta$ Cnc |
|  |  |  |  |  |  | 07h 17m | 2349 |  |  |  | M44 Praesepe - a better choice since mag is not defined; dec is given as apratima (maximum 23.5) |
|  | 106 | 0 |  | 08h 13m | 2034 | 08h 16m | 1951 |  |  |  | $\delta$ Cnc Padmanabha's values |
|  | 106 | 0 |  | 07h 10m | 2313 |  |  |  |  |  | SS Values S\&L identify $\delta$ Cnc |
| Aslesha | 11230 | -6 55 | 3 | 07h 36m | 1446 | 07h 40m | 1618 |  |  |  | $\alpha$ Cnc identified but error in dec |
|  |  |  |  |  |  | 07h 31m | 1034 |  |  |  | $\varepsilon$ Hya as suggested by S\&L also does not match |
|  | 10714 | -6 53 |  | 08h 19m | 1325 | 08h 31m | 1340 |  |  |  | $\alpha$ Cnc Padmanabha AD 1428 |
| Magha | 12850 | 010 | 1 | 08h 46m | 1317 | 08h 51m | 1806 |  |  |  | $\alpha$ Leo - error of $5^{\circ}$ |
|  | 10714 | 0 |  | 09h 46m | 1357 | 09h 41m | 1418 |  |  |  | $\alpha$ Leo Padmanabha's values (AD 1428) |
| Malayendu's | 141/23/52 | 010 |  |  |  |  | 13/39/19 | 75/1 | $\begin{aligned} & 4 / 25 / 11 / 28 \\ & 4 / 21 / 11 / 28 \end{aligned}$ | 1342 | Dhruvaka 145/11/28; in brackets 141/11/28 Name Kalupubhatrasada matches with Qalb al Asad |
| Pu Phalguni | 14215 | 930 | 3 | 09h 39m | 2349 | 09h 39m | 2241 |  |  |  | $\theta$ Leo |
|  | 14758 | 12 |  | 10h 01m | 2433 |  |  |  |  |  | $\theta$ Leo Padmanabha's values (AD 1428) |
| Ut Phalguni | 15024 | 12 | 1 | 10h 10m | 24 | 10h 35m | 2212 |  |  |  | $\beta$ Leo |
|  | 15530 | 1247 |  | 10h 37m | 2247 |  |  |  |  |  | $\beta$ Leo Padmanabha's values (AD 1428) |
| Malayendu's | 163/23/52 | 1150 |  | 10h 59m | 1823 |  | 16/15/37 | 78/6 | 5/17/31/50 | 1645 | Malayendu's values |

Table 3. (Contd)

the angular separation has changed over these centuries.

## The technique of measurement

All the texts available for study describe instruments used for measurement; however none describes the actual procedure. This has been criticized heavily by all scholars. With the simple techniques available for naked eye measurements, Saha and Lahiri ${ }^{5}$, Chandra Hari ${ }^{12}$, Abhyankar ${ }^{2}$ have deduced the probable procedure, which basically employs the noon shadow measurement. The instruments they had were very simple table top devices for example a $12^{\prime \prime}$ gnomon, which can lead to error of $1^{\circ}$ for the noon transit measures of the sun. In the case of stars, an armillary sphere with a sighting tube would yield slightly better results.

The procedure (Chandra Hari ${ }^{12}$ ) involved in noting the time and shadow length at noon. This fixes the declination of the sun. After dusk the meridian passage of the bright star is noted. This gives the declination of the star. Simultaneously the reading on the great circle joining the ecliptic pole to the equator also is noted. This gives the Vikśepa, which will be slightly different from the declination. The interval between the meridian passage of the sun and the star will give Dhruvaka.

It is not clear as to how the time interval was measured; it can be shown that it was not essential since the angle corresponding to Dhruvaka can be read out from a standard device like the armillary sphere.

Here is an example:
Consider the measurement being done from Bengaluru (latitude $13^{\circ}$ ) on July 29th. The meridian passage of the sun occurred at 12:28 IST and the altitude was $84^{\circ}$. This gives the declination of the sun as $+19^{\circ}$. Using this we can find the longitude $\lambda$ since

$$
\sin \delta=\sin \lambda \sin \varepsilon
$$

which gives longitude $\lambda$, of the sun as $128^{\circ}$ ( $\varepsilon$ is the obliquity of the ecliptic).

In the evening we measure the altitude at meridian transit for Antares (Jyeshta) occurring at 20:23 IST and the altitude is $50.5^{\circ}$.

Therefore, the declination, $\delta$, of Antares is $-26.5^{\circ}$.

The astrolabe has also the ecliptic marked. The angle read out from that scale was $-5^{\circ}$, which is the Viksepa, $\beta^{*}$.

Vikśepa and declination are related by

$$
\delta=\beta^{*}-\sin ^{-1}\left\{\sin \lambda^{*} \sin \varepsilon\right\},
$$

which gives the Dhruvaka, $\lambda^{*}$, as $-63^{\circ}$ or $243^{\circ}$ or $357^{\circ} ; 243^{\circ}$ suits here.

Alternately the time interval between the meridian passages of the sun and Antares can be used for getting the Dhruvaka. The measured interval is 7 : 55 which gives the angular separation as $120^{\circ}$. This is to be added to the longitude of the sun, $128^{\circ}$. Therefore Dhruvaka of Antares is $248^{\circ}$.

As can be seen there is an error in the calculation using the time difference, because the time difference of meridian passages will correspond to difference in right ascension and not Dhruvaka. (Dhruvaka is measured along the ecliptic.) This has been pointed out by Plofker ${ }^{13}$.

An error of $1^{\circ}$ in Vikśepa will result in almost $4^{\circ}$ for Dhruvaka.
The same technique is described by Middleton ${ }^{13}$. He describes a plate resembling an astrolabe which was used for conversions.
The errors in the measurements have been pointed out by earlier authors. The observational record of paramónnatámśa, which the maximum altitude (meridian passage) can at best be accurate to $0^{\circ} .5$ and the other values are derived from calculations. On the other hand with an astrolabe like instrument, Dhruvaka also can be read out. A clue to this comes from the study of Plofker ${ }^{14}$, who pointed out that the observers approximated the Vikśepa itself as declination for planets, which are generally close to the plane of ecliptic (therefore the errors are small).
There is no clue on the type of devices that were used prior to the introduction of astrolabe and therefore on how the correction to be applied for precession was incorporated on the dials. Aśvini is identified as $\gamma$ Ari by Pingree ${ }^{4}$ and others have identified as $\beta$ Ari. This confusion arises basically because of the Dhruvaka measurements. Samanta Chandrashekhar calls Aśvini as zero (origin for longitude measures, the First Point of Aries) and measures the Dhruvaka and Vikśepa of the yogatara named Aśvini. Saha and Lahiri ${ }^{5}$ considered the epoch of 1950 and converted all measurements to this
epoch. Such an extension is not possible for the values provided by Samantha. It is better to compare the values of declination rather than the right ascension.
Hipparcos has made precise measurements of the proper motion and parallax of all these stars. The approximate epoch of measurements can be read out from the text and the declination may be extrapolated. Here is the example for Rohini (Aldeberan). The measurements by Malayendu gives Dhruvaka as $2^{\mathrm{R}}$ $\left(2^{\mathrm{R}}=\right.$ two zodiacal constellations $\left.=60^{\circ}\right)$ $1^{\circ} 22^{\prime} 52^{\prime \prime}$, which means $61^{\circ} 22^{\prime} 52^{\prime \prime}$. He also provides Drkshuddha (corrected from observation) as $2^{\mathrm{R}} 2^{\circ} 36^{\prime} 13^{\prime \prime}$, which means $62^{\circ} 36^{\prime} 13^{\prime \prime}$. Using this value we can calculate the declination as $15^{\circ} 12^{\prime}$. The declination in the table is $15^{\circ} 22^{\prime} 7^{\prime \prime}$. From Hipparcos data this declination applied to 14th century which agrees within the observational errors for the date of the work.
Nityananda lists the values of Dhruvaka which are corrected for precession, in case of three stars specifically mentions ayanakarmayukta.

## Discussion

We have now a list of the 28 stars as obtained from different texts with the coordinates fixed from the declination value.

The procedure for estimating the declination has been described in great detail by Plofker ${ }^{14}$. The approximations to be done for smaller values of Vikśepa have been pointed out. However, such approximations are applicable case of planets, as their latitudes are small; but not for stars of higher latitudes. This becomes especially obvious in the case of Sirius.

Although Mahendra Suri was the first to write a book on the astrolabe, the values provided by him for Dhruvaka and Vikśepa seem to be in great error; they were simply reproduced from original Islamic source which in turn agree with the values provided by Ptolemy (Ohashi ${ }^{7}$ ). Later, Malayendu Suri provided the commentary and tabulated the coordinates. By comparing these values, we may infer that they were derived from actual measurements.
It appears that not much weightage was given to these tables. Subsequent authors continued to provide the coordinates from Surya Siddhántha. This


Figure 3. The (silver) instrument used in Kota during the 19th century, described by Middleton, reproduced from the Journal of the Asiatic Society.
makes it difficult to fix the epoch or even cross check with the actual values.

Most of the subsequent books concentrated on the movement of planets, prediction of eclipses and phenomenon related to planets. Extension to other stars, though not many, is in progress with a few star tables available for comparison.

As discussed above, the book Siddhántha Rája by Nityananda provides coordinates of stars based on observation. Ohashi has studied Yantra Kiranávali by Padmanabha and provides the Dhruvaka and Vikśepa of stars, also based on observations.

Almost all the texts provide the coordinates for Lubdhaka (Sirius) and Agastya (Canopus); it is puzzling that the values of Vikśepa are more or less the same in all cases irrespective of the epoch, while in the case of Dhruvaka there is a variation. This can be interpreted in two ways: (1) They followed the method of measuring the angles at the meridian passage; the values of Vikśepa have observational errors; (2) Alternately they used the time difference method for measuring Dhruvaka but did not correct it for precession.

Nityananda clearly mentions that he is citing the coordinates of Agastya (he states - knowledgeable people state so); perhaps the star was not visible for him to make actual measurements from his place Indrapuri, (as mentioned by him at the end of the text).
Abhyankar pointed out that Dhruvaka and Vikśepa system is independent of the epoch, giving a clue on the instrument that might have been used for this purpose. An instrument like the Kránti Vritta with graduation along the ecliptic reads out the Dhvarka. As it was only the difference that mattered for calculations, the epoch being fixed otherwise, the readings taken on the face value seem to be in error.

This resolves the ambiguity in the coordinates which can be used for extrapolated calculations; the residuals are experimental errors, as can be tolerated of a small table top instrument.

It remains a question as to why the tables continued with the values of Surya Siddhántha epoch. One possible reason may be inferred by a silver instrument in Kota has been described by Middleton ${ }^{13}$ and is reproduced in Figure 3. Such inherited instruments had these numbers
(Dhruvakas-vikśepas and/or longitudeslatitudes) engraved. Conversion of these readings to the current date was perhaps common knowledge and not explicitly mentioned in the texts.
Karañapaddhathi lists the longitudes as the double the actual values. One of the reasons may be because the angles were measured from a device which has to be viewed the reflection from a water surface. Such a technique has been described in Siddhántha Śẻkhara by Sripathi in the 11 th century (Bhat ${ }^{15}$ ). Incidentally, this text also provides Dhruvakas of the 27 stars; the values are the same as these mentioned in Surya Siddhántha.

## Conclusions

The coordinates of stars as listed in various astronomical texts have been studied with the purpose of understanding the techniques that were devised for measurements. Most of the earlier attempts were based on extrapolating the precession corrections, leading to confusions. This work described the actual observational aspects and possible reasons for
retaining old values for tabulation. In the process, it also revealed the finer observational details like estimates of brightness measures even for nebulosities. A comparison with coordinates from Stellarium showed the refraction and other instrumental effects contributing to the errors.

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