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# Oblong Mean Prime Labeling and Oblong Difference Mean Prime Labeling of Complete Graphs and Complete Multipartite Graphs

**P. Hemalatha* and V. Sudha**

Department of Mathematics, Vellalar College for Women, Erode – 638 012, Tamil Nadu, India; dr.hemalatha@gmail.com, sudha.vmath@gmail.com

## Abstract

The oblong numbers are in the form n(n+1), where n = 1,2, . . . . i.e., the oblong numbers are 2, 6, 12, . . . . If the vertices of the given graph G are labeled with oblong numbers and the edges of the graph are labeled with mean of the labels at the end vertices then G is said to have Oblong Mean Prime Labeling (OMPL). Similarly, if the vertices of G are labeled with oblong numbers and the edges of the graphs are labeled with mean of the absolute difference of the labels at the end vertices then G is said to have Oblong Difference Mean Prime Labeling (ODMPL). In this paper, the Oblong Mean Prime Labeling and Oblong Difference Mean Prime Labeling of Complete Graphs (CGs) $K_n$, $n \geq 3$ and Complete Multipartite Graphs (CMGs), $K_{n_1,n_2,\ldots,n_t}$, $n_i \geq 1$ where $1 \leq i \leq t$ have been investigated and obtained the results for such graphs.

**Keywords:** Complete Graphs (CGs) and Complete Multipartite Graphs (CMGs), Oblong Difference Mean Prime Labeling(ODMPL), Oblong Mean Prime Labeling (OMPL)

## 1. Introduction

Graph theory was first introduced by Leonhard Euler in the year 1736. He considered the Konigsberg bridge problem as a graph model and found that there is no solution to this problem. Graph labeling was first introduced by Rosa in 1967. In 2017, Mathew Varkey T. K. and Sunoj B. S. introduced the concept of Oblong Mean Prime Labeling and studied the existence of Oblong Mean Prime Labeling for cycles[4] and snake[6] graphs. In 2018, M. Prema and K. Murugan developed Oblong Mean Prime Labeling of some graphs. In 2018, Mathew Varkey T. K. and Sunoj B. S. have studied the Oblong Mean Prime Labeling of trees and planar graphs. In 2018, Mathew Varkey T. K. and Sunoj B. S. studied the existence of introduced the Oblong Difference Mean Prime Labeling and obtained the results for the existence of Oblong Difference Mean Prime Labeling for cycles[4].

In this paper, we have studied the concept of Oblong Mean Prime Labeling and Oblong Difference Mean Prime Labeling for Complete Graphs (CG) and Complete Multipartite Graphs (CMG).

## 2. Preliminaries

### 2.1 Definition 2.1[1]

A **Complete Graph** (CG) is a simple graph in which each pair of distinct vertices is joined by an edge is called. The CG on *n* vertices is denoted by $K_n$.

### 2.2 Definition 2.2[2]

A graph $G$ is $t$ – partite if its vertex set can be partitioned into $t$ independent sets, $V_1$, $V_2$,.., $V_t$ such that $|V_i| = n_i \geq 1$ for all $1 \leq i \leq t$ and every vertex in $V_i$ is adjacent to every

*Author for correspondence

vertex in $V_j$ whenever $i \neq j$. Then $V_1, V_2, \ldots, V_t$ are called partite sets of *G* and *G* is said to be a complete multipartite graph (CMG). The Complete Multipartite Graph on *t* partite set is denoted by $K_{n_1, n_2, \ldots, n_t}$. If $t = 2$, then *G* is a complete bipartite graph and if $t = 3$, then *G* is a Complete Tripartite Graph.

### 2.3 Definition 2.3[3]

The assignment of integers to the vertices or edges (or both) subject to certain conditions is called labeling.

### 2.4 Definition 2.4[8]

The Greatest Common Divisor (GCD), also known as the Greatest Common Denominator, Greatest Common Factor (GCF), or Highest Common Factor (HCF), of two or more non-zero integers, is the largest $+^{ve}$ integer that divides the numbers without a remainder.

### 2.5 Definition 2.5[4]

Let *G* be a graph with *m* vertices and *n* edges such that $d(v_i) \geq 2$, for i = 1, 2, . . ., *m*. The Greatest Common Incidence Number (GCIN) of a vertex $v_i$ is the Greatest Common Divisor of the labels of the edges incident at $v_i$.

### 2.6 Definition 2.6[3]

The product of a number with its successor is called an Oblong Number, algebraically it has the form *n(n+1)* where $n \in N$. The oblong numbers are 2,6,12,20, . . .

### 2.7 Definition 2.7[3]

Let *G* be a graph with *m* vertices and *n* edges. Define a bijection *f*: $V(G) \rightarrow \{2, 6, 12, \ldots, m(m+1)\}$ by $f(v_i)$ = i(i+1), for every i from 1 to m and define a 1-1 mapping $f^*_{odmpl} = \left|\frac{f(u)-f(v)}{2}\right|$. The induced function $f^*_{ompl}$ is said to be an **OMPL**, if the GCIN of each vertex of degree at least 2, is one. A graph which admits an Oblong Mean Prime Labeling is called an Oblong Mean Prime Graph (OMPG).

### 2.8 Definition 2.8[5]

Let *G* be a graph with *m* vertices and *n* edges. Define a bijection f: $V(G) \rightarrow \{2, 6, 12, \ldots, m(m+1)\}$ by $f(v_i)$ = i(i+1), for every i from 1 to *m* and define a 1-1 mapping $f^*_{odmpl} = \left|\frac{f(u)-f(v)}{2}\right|$. The induced function $f^*_{ompl}$ is said to be an ODMPL, if the GCIN of each vertex of degree at least 2, is one. A graph which admits an oblong difference mean prime labeling is called an Oblong Difference Mean Prime Graph (ODMPG).

## 3. Oblong Mean Prime Labeling of Graphs

### 3.1 Oblong Mean Prime Labeling of CGs

**Theorem 3.1.1:** For all $+^{ve}$ integers n≥ 3, the graph $K_n$ admits OMPL.

**Proof:** Let $= K_n$, $n \geq 3$ & let $v_1, v_2, \ldots, v_n$ be the vertices of *G*.

Here, $|V(G)| = n$ and $|E(G)| = \frac{n(n-1)}{2}$

Define a function $f : V \rightarrow \{2, 6, 12, \ldots, p(p+1)\}$ by

$f(v_i) = i(i+1)$, i = 1, 2, . . ., n.

Clearly f is a bijection.

For the vertex labeling f, $f^*_{ompl}$ is defined as follows:

$$f^*_{ompl}(v_i v_{i+1}) = (i+1)^2, i = 1, 2, \ldots, n-1$$

$$f^*_{ompl}(v_i v_n) = \frac{n^2+n+2}{2}$$

$$f^*_{ompl}(v_1 v_i) = \frac{i^2+i+2}{2}, \ i = 3, 4, \ldots, n-1$$

$$f^*_{ompl}(v_2 v_i) = \frac{i^2+i+6}{2}, \ i = 4, 5, \ldots, n$$

$$f^*_{ompl}(v_3 v_i) = \frac{i^2+i+12}{2}, \ i = 5, 6, \ldots, n$$

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$$f^*_{ompl}(v_{n-2} v_n) = \frac{n^2+n+(n-2)^2+(n-2)}{2}, \ i = 3, 4, \ldots, n-1$$

Clearly $f^*_{ompl}$ is an one-one.

The *gcin* of each vertex $v_i$ is defined as follows:

Gcin $(v_{i+1}) = \gcd(f^*_{ompl}(v_i v_{i+1}), f^*_{ompl}(v_{i+1} v_{i+2}))$, i=1, 2, . . . , n − 2

= gcd ((i+1)², (i+2)²)
= gcd ((i+1), (i+2))
= 1

$\text{gcin }(v_1) = \gcd(f^*_{ompl}(v_1v_2), f^*_{ompl}(v_1v_3)),$

$= \gcd\ (f^*_{ompl}(v_1v_2), f^*_{ompl}(v_1v_3))$

= gcd (4, 7)
= 1

$\text{gcin }(v_n) = \gcd(f^*_{ompl}(v_1v_n), f^*_{ompl}(v_3v_n)),$

$= \gcd\ \left(\frac{n^2+n+2}{2}, \frac{n^2+n+12}{2}\right)$

= 1

So Greatest Common Incidence Number (GCIN) of each vertex of degree greater than one is 1.
Hence the graph $K_n$, $n \geq 3$ admits an OMPL.

**Example:**

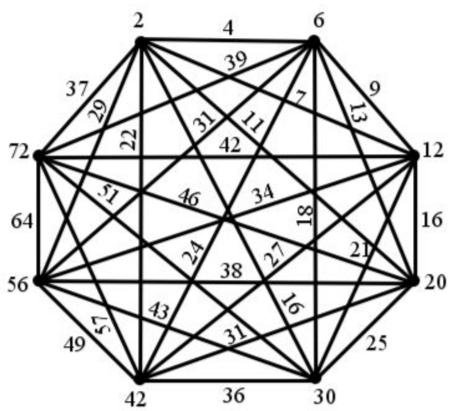


**Figure 3.1.1:** Oblong Mean Prime Labeling of $K_8$

## 3.2 Oblong Mean Prime Labeling of CMGs

**Theorem 3.2.1:** For all $+^{ve}$ integers $n_1, n_2, \ldots, n_t \geq 1$, the graph $K_{n_1,n_2,\ldots,n_t}$ OMPL.

**Proof:** Let $G = K_{n_1,n_2,\ldots,n_t}$, where $n_1, n_2, \ldots, n_t \geq 1$ & $V_1, V_2, \ldots, V_n$ be the t partite sets of G and let $v_{i1}, v_{i2}, \ldots, v_{in_i}$ be the vertices of $V_i$, i = 1, 2, . . .,t.

Here, $|V(G)| = n_1 + n_2 + \ldots + n_t = k$ and

$$|E(G)| = n_1(n_2+n_3+\ldots+n_t) + n_2(n_3+n_4+\ \ldots\ +n_t) + \ldots + n_{(t-1)}(n_t)$$

Define a function $f : V \to \{2, 6, 12, \ldots, p(p+1)\}$ by

$f(v_{1i}) = i(i+1)$, i = 1, 2, . . ., $n_1$

$f(v_{2i}) = (n_1+i)(n_1+i+1)$, i = 1, 2, . . ., $n_2$

$f(v_{3i}) = (n_1+n_2+i)(n_1+n_2+i+1)$, i = 1, 2, . . ., $n_3$

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$$f(v_{ti}) = (n_1+\ldots+n_{(t-1)}+i)(n_1+\ldots+n_{(t-1)}+i+1),$$

i = 1, 2, . . ., $n_t$

Clearly f is a bijection.
For the vertex labeling f, $f^*_{ompl}$ is defined as follows:

$$f^*_{ompl}(v_{1i}v_{2j}) = \frac{i^2+i+(n_1+j)^2+(n_1+j)}{2},$$

$i = 1, 2, \ldots, n_1$ & $j = 1, 2, \ldots, n_2$

$$f^*_{ompl}(v_{1i}v_{3j}) = \frac{i^2+i+(n_1+n_2+j)^2+(n_1+n_2+j)}{2},$$

$i = 1, 2, \ldots, n_1$ & $j = 1, 2, \ldots, n_3$

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$$f^*_{ompl}v_{1i}v_{tj}) = \frac{i^2+i+(n_1+n_2+\ldots+n_{(t-1)}+j)^2+(n_1+n_2+\ldots+n_{(t-1)}+j)}{2},$$

$i = 1, 2, \ldots, n_1$ & $j = 1, 2, \ldots, n_t$

$$f^*_{ompl}(v_{2i}v_{3j}) = \frac{(3+i)^2+(3+i)+(n_1+n_2+j)^2+(n_1+n_2+j)}{2},$$

$i = 1, 2, \ldots, n_2$ & $j = 1, 2, \ldots, n_3$

$$f^*_{ompl}(v_{2i}v_{4j}) = \frac{(3+i)^2+(3+i)+(n_1+n_2+n_3+j)^2+(n_1+n_2+n_3+j)}{2},$$

$i = 1, 2, \ldots, n_2$ & $j = 1, 2, \ldots, n_4$

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$$f^*_{ompl}(v_{2i}v_{tj}) = \frac{(3+i)^2+(3+i)+\ (n_1+\ldots+n_{(t-1)}+j)^2+(n_1+\ldots+n_{(t-1)}+j)}{2}$$

$i = 1, 2, \ldots, n_2$ & $j = 1, 2, \ldots, n_t$

$$f^*_{ompl}(v_{3i}v_{4j}) = \frac{(5+i)^2+(5+i)+(n_1+n_2+n_3+j)^2+(n_1+n_2+n_3+j)}{2},$$

$i = 1, 2, \ldots, n_3$ & $j = 1, 2, \ldots, n_4$

$$f^*_{ompl}(v_{3i}v_{5j}) =$$

$$\frac{(5+i)^2+(5+i)+(n_1+n_2+n_3++n_4+j)^2+(n_1+n_2+n_3++n_4+j)}{2} i = 1, 2, \ldots, n_3$$

& $j = 1, 2, \ldots, n_5$

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$f^*_{ompl}\left(v_{3i}v_{tj}\right)$

$$=\frac{(5+i)^2+(5+i)+(n_1+\ldots+n_{(t-1)}+j)^2+\left(n_1+\ldots+n_{(t-1)}+j\right)}{2}$$

$i=1,2,\ldots,n_3$ & $j=1,2,\ldots,n_t$

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$f^*_{ompl}\left(v_{(t-1)i}v_{tj}\right)$

$$=\frac{(2t-3+i)^2+(2t-3+i)+(n_1+\ldots+n_{(t-1)}+j)^2+\left(n_1+\ldots+n_{(t-1)}\right)}{2}$$

$i=1,2,\ldots,n_{(t-1)}$ & $j=1,2,\ldots,n_t$

Clearly $f^*_{ompl}$ is an one-one.
The gcin of each vertex $v_{ij}$ is defined as follows:

$$\text{gcin}(v_{1i})=\gcd(f^*_{ompl}\left(v_{1i}v_{(t-1)1}\right), f^*_{ompl}\left(v_{1i}v_{(t-1)2}\right), f^*_{ompl}\left(v_{1i}v_{(t-1)3}\right), f^*_{ompl}\left(v_{1i}v_{(t-1)4}\right), f^*_{ompl}\left(v_{1i}v_{t1}\right), f^*_{ompl}\left(v_{1i}v_{t2}\right), f^*_{ompl}\left(v_{1i}v_{t3}\right), f^*_{ompl}\left(v_{1i}v_{t4}\right)$$

$i=1,2,\ldots,n_1$

$$=\gcd(\frac{i^2+i+(n_1+n_2+\ldots+n_{(t-2)}+1)^2+\left(n_1+n_2+\ldots+n_{(t-2)}+1\right)}{2},$$
$$\frac{i^2+i+(n_1+n_2+\ldots+n_{(t-2)}+2)^2+\left(n_1+n_2+\ldots+n_{(t-2)}+2\right)}{2},$$
$$\frac{i^2+i+(n_1+n_2+\ldots+n_{(t-2)}+3)^2+\left(n_1+n_2+\ldots+n_{(t-2)}+3\right)}{2},$$
$$\frac{i^2+i+(n_1+n_2+\ldots+n_{(t-2)}+4)^2+\left(n_1+n_2+\ldots+n_{(t-2)}+4\right)}{2},$$
$$\frac{i^2+i+(n_1+n_2+\ldots+n_{(t-1)}+1)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+1\right)}{2},$$
$$\frac{i^2+i+(n_1+n_2+\ldots+n_{(t-1)}+2)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+2\right)}{2},$$
$$\frac{i^2+i+(n_1+n_2+\ldots+n_{(t-1)}+3)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+3\right)}{2},$$
$$\frac{i^2+i+(n_1+n_2+\ldots+n_{(t-1)}+3)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+4\right)}{2})$$

$= 1$

$$\text{gcin}(v_{ij})=\gcd(f^*_{ompl}\left(v_{ij}v_{11}\right), f^*_{ompl}\left(v_{ij}v_{12}\right), f^*_{ompl}\left(v_{ij}v_{13}\right), f^*_{ompl}\left(v_{ij}v_{14}\right), f^*_{ompl}\left(v_{ij}v_{t1}\right), f^*_{ompl}\left(v_{ij}v_{t2}\right), f^*_{ompl}\left(v_{ij}v_{t3}\right), f^*_{ompl}\left(v_{ij}v_{t4}\right))$$

$i=2,\ldots,t-1$ & $j=1,2,\ldots,n_i$

$$=\gcd(\frac{(2i-1+j)^2+(2i-1+j)+2}{2},$$
$$\frac{(2i-1+j)^2+(2i-1+j)+6}{2},$$
$$\frac{(2i-1+j)^2+(2i-1+j)+12}{2},$$
$$\frac{(2i-1+j)^2+(2i-1+j)+20}{2},$$
$$\frac{(2i-1+j)^2+(2i-1+j)+(n_1+n_2+\ldots+n_{(t-1)}+1)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+1\right)}{2},$$
$$\frac{(2i-1+j)^2+(2i-1+j)+(n_1+n_2+\ldots+n_{(t-1)}+2)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+2\right)}{2},$$
$$\frac{(2i-1+j)^2+(2i-1+j)+(n_1+n_2+\ldots+n_{(t-1)}+3)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+3\right)}{2},$$
$$\frac{(2i-1+j)^2+(2i-1+j)+(n_1+n_2+\ldots+n_{(t-1)}+4)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+4\right)}{2})$$

$= 1$

$$\text{gcin}(v_{ti})=\gcd(f^*_{ompl}\left(v_{ti}v_{11}\right), f^*_{ompl}\left(v_{ti}v_{12}\right), f^*_{ompl}\left(v_{ti}v_{13}\right), f^*_{ompl}\left(v_{ti}v_{14}\right), f^*_{ompl}\left(v_{ti}v_{21}\right), f^*_{ompl}\left(v_{ti}v_{22}\right), f^*_{ompl}\left(v_{ti}v_{23}\right), f^*_{ompl}\left(v_{ti}v_{24}\right))$$

$i=1,2,\ldots,n_t$

$$= \gcd\Big( \frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)+2}{2},$$
$$\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)+6}{2},$$
$$\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)+12}{2},$$
$$\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)+20}{2},$$
$$\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)+(n_1+1)^2+\left(n_1+1\right)}{2},$$
$$\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)+(n_1+2)^2+\left(n_1+2\right)}{2},$$
$$\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)+(n_1+3)^2+\left(n_1+3\right)}{2},$$
$$\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)+(n_1+4)^2+\left(n_1+4\right)}{2}\Big)$$

$= 1$

So Greatest Common Incidence Number (GCIN) of each vertex of degree greater than one is 1.

Hence the graph $K_{n_1,n_2,\ldots,n_t}$, where $n_1, n_2, \ldots, n_t \geq 1$ admits an OMPL.

**Example:**

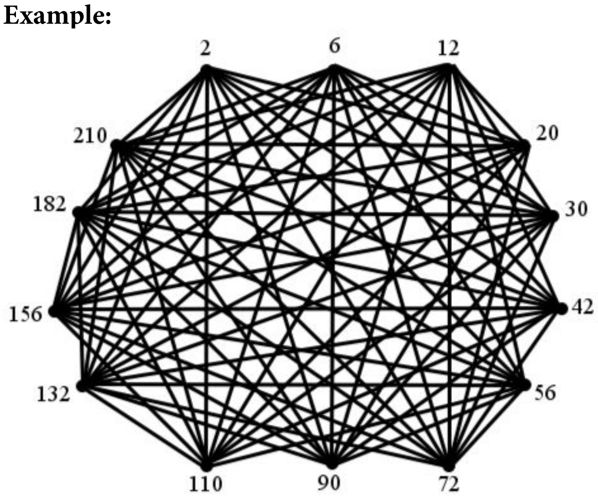


**Fig 3.1.2:** Oblong Mean Prime Labeling of $K_{3,2,2,3,2,2,}$

**Corollary 3.2.1:** Let $n_1$ & $n_2$ be $+^{ve}$ integers such that $n_1 < n_2$. Then

(i) $K_{1,n_2}$ has OMPL for $n_2 \geq 2$

(ii) $K_{n_1,n_2}$ has OMPL for $n_1, n_2 \geq 3$.

**Proof:**

(i) Let $G = K_{1,n_2}$, where $n_2 \geq 2$.

Let $V_1$ & $V_2$ be the two partite sets of *G* such that $|V_1| = 1$ & $|V_2| = n_2$.

Let $v_{11}$ be the vertex of $V_1$ & $v_{2j}$, $j = 1, 2, \ldots, n_2$ be the vertices of $V_2$.

Here, $|V(G)| = 1 + n_2$ and $|E(G)| = n_2$.

For OMPL the vertex labeling and edge labeling are defined as in Theorem 3.2.1.

The Greatest Common Incidence Number is defined as follows:

$gcin(V_{11}) = \gcd\left(f^*_{ompl}\left(v_{11}v_{21}\right), f^*_{ompl}\left(v_{11}v_{22}\right)\right)$,

$= \gcd(4, 7)$

$= 1$

Then $K_{1,n_2}$ has Oblong Mean Prime Labeling.

(ii) If $n_1, n_2 \geq 3$ the OMPL of $K_{n_1,n_2}$ follows from Theorem 3.2.1.

# 4. Oblong Difference Mean Prime Labeling of Graphs

## 4.1 Oblong Difference Mean Prime Labeling of Complete Graphs

**Theorem 4.1.1:** For all $+^{ve}$ integers n ≥ 3, the graph $K_n$ admits ODMPL.

**Proof:** Let $G = K_n$, n ≥ 3 and let $v_1, v_2, \ldots, v_n$ be the vertices of *G*.

Here, $|V(G)| = n$ and $|E(G)| = \dfrac{n(n-1)}{2}$

Define a function $f : V \to \{2, 6, 12, \ldots, p(p+1)\}$ by $f(v_i) = i(i+1)$, i = 1, 2, . . ., n.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{ompl}$ is defined as follows:

$f^*_{odmpl}\left(v_i v_{i+1}\right) = (i+1)$, i = 1, 2, . . ., n-1

$$f^*_{odmpl}(v_i v_n) = \left|\frac{n^2+n-2}{2}\right|$$

$$f^*_{odmpl}(v_1 v_i) = \left|\frac{i^2+i-2}{2}\right|, i = 3, 4, \ldots, n\text{-}1$$

$$f^*_{odmpl}(v_2 v_i) = \left|\frac{i^2+i-6}{2}\right|, i = 4, 5, \ldots, n$$

$$f^*_{odmpl}(v_3 v_i) = \left|\frac{i^2+i-12}{2}\right|, i = 5, 6, \ldots, n$$

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$$f^*_{odmpl}(v_{n-2} v_n) = \left|\frac{n^2+n+(n-2)^2+(n-2)}{2}\right|,$$

i = 3, 4, . . ., n-1

Clearly $f^*_{ompl}$ is an one-one.

The Greatest Common Incidence Number (GCIN) of each $v_i$ is defined as follows:

gcin($V_{i+1}$) = gcd ( $f^*_{odmpl}(v_i v_{i+1})$, $f^*_{odmpl}(v_{i+1} v_{i+2})$ ),

i = 1, 2, . . ., n-2

= gcd ( (i+1), (i+2))

= 1

gcin ($v_1$) = gcd ( $f^*_{odmpl}(v_1 v_2)$, $f^*_{odmpl}(v_1 v_3)$ ),

= gcd(4, 7)

= 1

gcin ($v_n$) = gcd ( $f^*_{odmpl}(v_1 v_n)$, $f^*_{odmpl}(v_3 v_n)$ ),

$$= \gcd\left(\left|\frac{n^2+n-2}{2}\right|, \left|\frac{n^2+n-12}{2}\right|\right)$$

= 1

So GCIN of each vertex of degree greater than one is 1.

Hence the graph $K_n$, n ≥ 3 admits ODMPL.

## 4.2 Oblong Difference Mean Prime Labeling of Complete Multipartite Graphs

**Theorem 4.2.1:** For all $+^{ve}$ integers $n_1, n_2, \ldots, n_t \geq 1$, the complete multipartite graph $K_{n_1, n_2, \ldots, n_t}$ admits an ODMPL.

**Proof:** Let $G = K_{n_1, n_2, \ldots, n_t}$, where $n_1, n_2, \ldots, n_t \geq 1$ & $V_1, V_2, \ldots, V_n$ be the t partite sets of G and let $v_{i1}, v_{i2}, \ldots, v_{in_i}$ be the vertices of $V_i$, i = 1, 2, . . .,t.

Here, |V(G)| = $n_1 + n_2 + \ldots + n_t$ = k and

$|E(G)| = n_1(n_2+n_3+\ldots+n_t) + n_2(n_3+n_4+\ldots n_t)$

$+ \ldots + n_{(t-1)}(n_t)$

Define a function $f: V \to \{2, 6, 12, \ldots, p(p+1)\}$ by

$f(v_{1i}) = i(i+1)$, i = 1, 2, . . ., $n_1$

$f(v_{2i}) = (n_1+i)(n_1+i+1)$, i = 1, 2, . . ., $n_2$

$f(v_{3i}) = (n_1+n_2+i)(n_1+n_2+i+1)$, i = 1, 2, . . ., $n_3$

.

.

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$$f(v_{ti}) = (n_1+\ldots+n_{(t-1)}+i)(n_1+\ldots+n_{(t-1)}\ i+1),$$

$i = 1, 2, \ldots, n_t$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling $f^*_{ompl}$ is defined as follows:

$$f^*_{odmpl}(v_{1i} v_{2j}) = \left|\frac{i^2+i-(n_1+j)^2-(n_1+j)}{2}\right|,$$

$i = 1, 2, \ldots, n_1$ & $j = 1, 2, \ldots, n_2$

$$f^*_{odmpl}(v_{1i} v_{3j}) = \left|\frac{i^2+i-(n_1+n_2+j)^2-(n_1+n_2+j)}{2}\right|,$$

$i = 1, 2, \ldots, n_1$ & $j = 1, 2, \ldots, n_3$

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$$f^*_{odmpl} v_{1i} v_{tj}) = \left|\frac{i^2+i-(n_1+n_2+\ldots+n_{(t-1)}+j)^2-(n_1+n_2+\ldots+n_{(t-1)}+j)}{2}\right|$$

$i = 1, 2, \ldots, n_1$ & $j = 1, 2, \ldots, n_t$

$$f^*_{odmpl}(v_{2i} v_{3j}) = \left|\frac{(3+i)^2+(3+i)-(n_1+n_2+j)^2-(n_1+n_2+j)}{2}\right|,$$

$i = 1, 2, \ldots, n_2$ & $j = 1, 2, \ldots, n_3$

$$f^*_{odmpl}\left(v_{2i}v_{4j}\right)=\left|\frac{(3+i)^2+(3+i)-(n_1+n_2+n_3+j)^2-(n_1+n_2+n_3+j)}{2}\right|,$$

$i=1,2,\ldots,n_2$ & $j=1,2,\ldots,n_4$

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$$f^*_{odmpl}\left(v_{2i}v_{tj}\right)=\left|\frac{(3+i)^2+(3+i)-(n_1+\ldots+n_{(t-1)}+j)^2-(n_1+\ldots+n_{(t-1)}+j)}{2}\right|,$$

$i=1,2,\ldots,n_2$ & $j=1,2,\ldots,n_t$

$$f^*_{odmpl}(v_{3i}v_{4j})=\left|\frac{(5+i)^2+(5+i)-(n_1+n_2+n_3+j)^2-(n_1+n_2+n_3+j)}{2}\right|,$$

$i=1,2,\ldots,n_3$ & $j=1,2,\ldots,n_4$

$$f^*_{odmpl}\left(v_{3i}v_{5j}\right)=\left|\frac{(5+i)^2+(5+i)-(n_1+n_2+n_3+n_4+j)^2-(n_1+n_2+n_3+n_4+j)}{2}\right|,$$

$i=1,2,\ldots,n_3$ & $j=1,2,\ldots,n_5$

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$$f^*_{odmpl}\left(v_{3i}v_{tj}\right)=\left|\frac{(5+i)^2+(5+i)-(n_1+\ldots+n_{(t-1)}+j)^2-(n_1+\ldots+n_{(t-1)}+j)}{2}\right|,$$

$i=1,2,\ldots,n_3$ & $j=1,2,\ldots,n_t$

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$$f^*_{odmpl}\left(v_{(t-1)i}v_{tj}\right)=\left|\frac{(2t-3+i)^2+(2t-3+i)-(n_1+\ldots+n_{(t-1)}+j)^2-(n_1+\ldots+n_{(t-1)}+j)}{2}\right|$$

$i=1,2,\ldots,n_{(t-1)}$ & $j=1,2,\ldots,n_t$

Clearly $f^*_{odmpl}$ is an injection.

The gcin of each vertex $v_{ij}$ is defined as follows:

$$\mathrm{gcin}(v_{1i}) = \gcd(f^*_{odmpl}\left(v_{1i}v_{(t-1)1}\right),\ f^*_{odmpl}\left(v_{1i}v_{(t-1)2}\right),$$

$$f^*_{odmpl}\left(v_{1i}v_{(t-1)3}\right),\ f^*_{odmpl}\left(v_{1i}v_{(t-1)4}\right)$$

$$f^*_{odmpl}\left(v_{1i}v_{t1}\right),\ f^*_{odmpl}\left(v_{1i}v_{t2}\right),$$

$$f^*_{odmpl}\left(v_{1i}v_{t3}\right),\ f^*_{odmpl}\left(v_{1i}v_{t4}\right),$$

$$i=1,2,\ldots,n_1=\gcd(\left|\frac{i^2+i-(n_1+n_2+\ldots+n_{(t-2)}+1)^2-(n_1+n_2+\ldots+n_{(t-2)}+1)}{2}\right|,$$

$$\left|\frac{i^2+i-(n_1+n_2+\ldots+n_{(t-2)}+2)^2-(n_1+n_2+\ldots+n_{(t-2)}+2)}{2}\right|,$$

$$\left|\frac{i^2+i-(n_1+n_2+\ldots+n_{(t-2)}+3)^2-(n_1+n_2+\ldots+n_{(t-2)}+3)}{2}\right|$$

$$\left|\frac{i^2+i-(n_1+n_2+\ldots+n_{(t-2)}+4)^2-(n_1+n_2+\ldots+n_{(t-2)}+4)}{2}\right|$$

$$\left|\frac{i^2+i-(n_1+n_2+\ldots+n_{(t-1)}+1)^2-(n_1+n_2+\ldots+n_{(t-1)}+1)}{2}\right|$$

$$\left|\frac{i^2+i-(n_1+n_2+\ldots+n_{(t-1)}+2)^2-(n_1+n_2+\ldots+n_{(t-1)}+2)}{2}\right|,$$

$$\left|\frac{i^2+i-(n_1+n_2+\ldots+n_{(t-1)}+3)^2-(n_1+n_2+\ldots+n_{(t-1)}+3)}{2}\right|$$

$$\left|\frac{i^2+i-(n_1+n_2+\ldots+n_{(t-1)}+3)^2-(n_1+n_2+\ldots+n_{(t-1)}+4)}{2}\right|)$$

$=1$

$$\mathrm{gcin}(v_{ij})=\gcd(f^*_{odmpl}\left(v_{ij}v_{11}\right),\ f^*_{odmpl}\left(v_{ij}v_{12}\right),$$

$$f^*_{odmpl}\left(v_{ij}v_{13}\right),\ f^*_{odmpl}\left(v_{ij}v_{14}\right),$$

$$f^*_{odmpl}\left(v_{ij}v_{t1}\right),\ f^*_{odmpl}\left(v_{ij}v_{t2}\right),$$

$$f^*_{odmpl}\left(v_{ij}v_{t3}\right),\ f^*_{odmpl}\left(v_{ij}v_{t4}\right))$$

$i=2,\ldots,t-1$ & $j=1,2,\ldots,n_I$

$$=\gcd(\left|\frac{(2i-1+j)^2+(2i-1+j)-2}{2}\right|,\left|\frac{(2i-1+j)^2+(2i-1+j)-6}{2}\right|,$$

$$\left|\frac{(2i-1+j)^2+(2i-1+j)-12}{2}\right|,\ \left|\frac{(2i-1+j)^2+(2i-1+j)-20}{2}\right|,$$

$$\left|\frac{(2i-1+j)^2+(2i-1+j)-(n_1+n_2+\ldots+n_{(t-1)}+1)^2-(n_1+n_2+\ldots+n_{(t-1)}+1)}{2}\right|$$

$$\left|\frac{(2i-1+j)^2+(2i-1+j)-(n_1+n_2+\ldots+n_{(t-1)}+2)^2-\left(n_1+n_2+\ldots+n_{(t-1)}+2\right)}{2}\right|,$$

$$\left|\frac{(2i-1+j)^2+(2i-1+j)-(n_1+n_2+\ldots+n_{(t-1)}+3)^2-\left(n_1+n_2+\ldots+n_{(t-1)}+3\right)}{2}\right|,$$

$$\left|\frac{(2i-1+j)^2+(2i-1+j)-(n_1+n_2+\ldots+n_{(t-1)}+4)^2-\left(n_1+n_2+\ldots+n_{(t-1)}+4\right)}{2}\right|)$$

= 1'

$$gcin(v_{ti}) = gcd\left( f^*_{odmpl} v_{ti} v_{11}\right),\ f^*_{odmpl}\left(v_{ti} v_{12}\right),$$

$$f^*_{odmpl}\left(v_{ti} v_{13}\right),\ f^*_{odmpl}\left(v_{ti} v_{14}\right),$$

$$f^*_{odmpl}\left(v_{ti} v_{21}\right),\ f^*_{odmpl}\left(v_{ti} v_{22}\right),$$

$$f^*_{odmpl}\left(v_{ti} v_{23}\right),\ \ f^*_{odmpl}\left(v_{ti} v_{24}\right)),$$

$$= gcd(\left|\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)-2}{2}\right|,$$

$$\left|\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)-6}{2}\right|,$$

$$\left|\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)-12}{2}\right|,$$

$$\left|\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)-20}{2}\right|$$

$$\left|\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)-(n_1+1)^2-\left(n_1+1\right)}{2}\right|,$$

$$\left|\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)-(n_1+2)^2-\left(n_1+2\right)}{2}\right|,$$

$$\left|\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)-(n_1+3)^2-\left(n_1+3\right)}{2}\right|,$$

$$\left|\frac{(n_1+n_2+\ldots+n_{(t-1)}+i)^2+\left(n_1+n_2+\ldots+n_{(t-1)}+i\right)-(n_1+4)^2-\left(n_1+4\right)}{2}\right|)$$

= 1

So Greatest Common Incidence Number (GCIN) of each vertex of degree greater than one is 1.

Hence the graph $K_{n_1,n_2,\ldots,n_t}$, where $n_1, n_2, \ldots, n_t \geq 1$ admits an ODMPL.

**Corollary 4.2.1:** Let $n_1$ and $n_2$ be positive integers such that $n_1 < n_2$. Then

(i) $K_{1,n_2}$ has ODMPL for $n_2 \geq 2$
(ii) $K_{n_1,n_2}$ has ODMPL for $n_1, n_2 \geq 3$

**Proof:**
(i) Let $G = K_{1,n_2}$, where $n_2 \geq 2$.

Let $V_1$ & $V_2$ be the two partite sets of *G* such that $|V_1| = 1$ & $|V_2| = n_2$.

Let $v_{11}$ be the vertices of $V_1$ & $V_{2j}$, $j = 1, 2, \ldots, n_2$ be the vertices of $V_2$.

Here,, $|V(G)| = 1 + n_2$ and $|E(G)| = n_2$.

The vertex labeling and edge labeling are defined as in Theorem 4.2.1.

The Greatest Common Incidence Number is defined as follows:

$$gcin(V_{11}) = gcd( f^*_{odmpl}\left(v_{11} v_{21}\right),\ f^*_{odmpl}\left(v_{11} v_{22}\right)),$$

= gcd( 2, 5 )

= 1

Then $K_{1,n_2}$ has ODMPL.

(ii) If $n_1, n_2 \geq 3$ the ODMPL of $K_{n_1,n_2}$ follows from Theorem 4.2.1.

## 5. Conclusion

In this paper, it is proved that the complete graphs $K_n$, $n \geq 3$ and complete multipartite graphs, $K_{n_1,n_2,\ldots,n_t}$, where $n_i \geq 1$, $1 \leq i \leq t$ admit both Oblong Mean Prime Labeling and Oblong Difference Mean Prime Labeling. Further, there is a scope for this labeling to be extended for some other classes of graphs.

## 6. References


1. Bondy JA, Murty USR. Graph Theory with Applications. New York: Elseveir Science Publishing Co., Inc; 1976. https://doi.org/10.1007/978-1-349-03521-2

2. Chih - Hung Yen. On r - Equitable Coloring of Complete Multipartite Graphs. Taiwanese Journal of Mathematics. 2013; 17(3):1–8. https://doi.org/10.11650/tjm.17.2013.2666
3. Gallian JA. A dynamic survey of graph labeling. The Electronic Journal of Combinatorics. 2009; 16:DS6.
4. Mathew Varkey TK, Sunoj BS. Oblong Mean Prime Labeling of some cycle graphs. Advances in Dynamical Systems and Applications. 2017; 12(2):181–6.
5. Mathew Varkey TK, Sunoj BS. Oblong Difference Mean Prime Labeling of some cycle graphs. International Journal of Statistics and Systems. 2017; 12(2):285–92.
6. Mathew Varkey TK, Sunoj BS. Oblong Mean Prime Labeling of some snake graphs. International Journal on Recent and Innovation Trends in Computing and Communication. 2017; 5(6):5–7.
7. Mathew Varkey TK, Sunoj BS. Oblong Difference Mean Prime Labeling of some cycle graphs. Anveshana's International Journal of Reseach in Engineering and Applied Sciences. 2018; 3(2):78–81.
8. Neville Robbins, Beginning Number eory, Narosa Publishing House Pvt. Ltd., Reprint 2006.