Abstract
In this paper, the theory of fuzzy semiprimary ideal [16] is extended by introducing intuitionistic anti fuzzy primary ideals as well as intuitionistic anti fuzzy semiprimary ideals in rings. Similarly, Interval-Valued Intuitionistic Anti Fuzzy Lie Primary Ideals (IVIAFLPI) is defined. Various properties of IVIAFLPI are discussed. Finally, Interval-Valued Intuitionistic Fuzzy Lie Semiprimary Ideals (IVIAFLSPI) is established.

Keywords: Intuitionistic Fuzzy Set, Intuitionistic Anti Fuzzy Ideal, Intuitionistic Anti Fuzzy Primary Ideal, Intuitionistic Anti Fuzzy Semi-Primary Ideal, Interval-Valued Intuitionistic Anti Fuzzy Lie Primary Ideals

1. Introduction
After the introduction of Fuzzy Sets (FSs) by A. Zadeh [12], the fuzzy concept has been used to extend almost all areas of mathematics. By using FSs people have recognized the theory to study uncertainty. Fuzzy mathematics have become a vital area of research in different applications such as engineering, medical science, social science, artificial intelligence, signal processing, pattern recognition, computer networks, automata theory and so on. The notion of IFS and its operations were introduced by Atanassov [1], as a generalization of the concept of FS. Atanassov [2] discussed the operators over Interval-Valued Intuitionistic Fuzzy Sets (IVIFSs). Palanivelrajan and Nandakumar [9] introduced the definition and some properties of intuitionistic fuzzy primary and semiprimary ideals [16].


In this paper, interval-valued intuitionistic anti fuzzy lie primary ideals and anti fuzzy Lie ideals, interval-valued intuitionistic anti fuzzy Lie ideals of Lie algebras are discussed.

2. Preliminaries
In this section, some basic definitions which are necessary for this paper are presented.

Definition 2.1 [11]
A fuzzy subset $\mu$ of a ring $R$ is called fuzzy ideal if for all $x, y \in R$ the subsequent conditions are satisfied
Some Properties of Interval-Valued Intuitionistic Anti Fuzzy Lie Primary Ideal

(i) \( \mu(x - y) \geq \min(\mu(x), \mu(y)) \)

(ii) \( \mu(xy) \geq \max(\mu(x), \mu(y)) \)

Example 2.1

\[ \mu = \begin{cases} 1, & \text{if } x = 0 \\ 0.8, & \text{if } x \in \{4\} \sim \{0\} \\ 0.6, & \text{if } x \in z \sim \{4\} \end{cases} \]

Take \( x = 4, y = 3, x - y = 1 \)

(i) \( \mu_A(x - y) \geq \min(\mu_A(x), \mu_A(y)) \)

(ii) \( \mu_A(xy) \geq \max(\mu_A(x), \mu_A(y)) \)

\[ \mu_A(1) \geq \min(\mu_A(4), \mu_A(3)) \]

\[ \mu_A(1) \geq \min(0.8, 0.6) \]

\[ 0.6 \geq 0.6 \]

\[ \mu_A(12) \geq \max(\mu_A(4), \mu_A(3)) \]

\[ \mu_A(12) \geq \max(0.8, 0.6) \]

\[ 0.8 \geq 0.8 \]

Definition 2.2 [14]

A fuzzy subset \( \mu \) of a ring \( R \) is called anti fuzzy ideal if for all \( x, y \in R \) the subsequent conditions are satisfied

(i) \( \mu(x - y) \leq \max(\mu(x), \mu(y)) \)

(ii) \( \mu(xy) \leq \min(\mu(x), \mu(y)) \).

Definition 2.3 [14]

A fuzzy subset \( \mu \) of a ring \( R \) is called intuitionistic anti fuzzy ideal if for all \( x, y \in R \) the subsequent conditions are satisfied,

(i) \( \mu_A(x - y) \leq \max(\mu_A(x), \mu_A(y)) \)

(ii) \( \mu_A(xy) \leq \min(\mu_A(x), \mu_A(y)) \)

(iii) \( \gamma_A(x - y) \geq \min(\gamma_A(x), \gamma_A(y)) \)

(iv) \( \gamma_A(xy) \geq \max(\gamma_A(x), \gamma_A(y)) \)

Example 2.2

Let \( R = \mathbb{Z} \), the ring of integers under ordinary addition and multiplication of integers.

Define the two IFS's \( A \) and \( B \) by

\[ \mu_A(x) = \begin{cases} 0.5, & \text{if } x \text{ is a multiple of } 3 \\ 1, & \text{otherwise} \end{cases} \]

\[ \mu_B(x) = \begin{cases} 0.8, & \text{if } x \text{ is a multiple of } 3 \\ 0.83, & \text{otherwise} \end{cases} \]

and

\[ \gamma_A(x) = \begin{cases} 0.3, & \text{if } x \text{ is a multiple of } 3 \\ 0, & \text{otherwise} \end{cases} \]

\[ \gamma_B(x) = \begin{cases} 0.15, & \text{if } x \text{ is a multiple of } 3 \\ 0.05, & \text{otherwise} \end{cases} \]

It can be easily verified that \( A \) and \( B \) are IAFI of \( \mathbb{Z} \).

Definition 2.4

An intuitionistic anti fuzzy ideal \( R \) of a ring \( R \) is called Intuitionistic anti fuzzy primary ideal (IAFPI) if for all \( a, b \in R \) either \( \mu_A(ab) = \mu_A(a) \) and \( \gamma_A(ab) = \gamma_A(a) \), or \( \mu_A(ab) \geq \mu_A(b^m) \) and \( \gamma_A(ab) \leq \gamma_A(b^n) \), for some \( m \in \mathbb{Z}^+ \).

Example 2.3

\[ \mu_A(x) = \begin{cases} 1, & \text{if } x = 0 \\ 0.8, & \text{if } x \in \{4\} \sim \{0\} \\ 0.6, & \text{if } x \in z \sim \{4\} \end{cases} \]

and

\[ \gamma_A(x) = \begin{cases} 0, & \text{if } x = 0 \\ 0.1, & \text{if } x \in \{4\} \sim \{0\} \\ 0.3, & \text{if } x \in z \sim \{4\} \end{cases} \]

Consider \( 0, 4 \in z \)

\[ \mu_A(0.4) = \mu_A(0) = 1 = 1 \]

\[ \gamma_A(0.4) = \gamma_A(0) = 0 = 0 \]

\[ \mu_A(3.4) = \mu_A(3) = 0.6 = 0.6 \]

\[ \gamma_A(3.4) = \gamma_A(3) = 0.3 = 0.3 \]

Now \( \mu_A(0.4) \geq \mu_A(4^m) \Rightarrow 1 \geq 0.8 \)

\[ \gamma_A(0.4) \leq \gamma_A(4^n) \Rightarrow 0 \leq 0.1 \]

\[ \mu_A(4.3) \geq \mu_A(3^n) \Rightarrow 0.6 \geq 0.6 \]

\[ \gamma_A(4.3) \leq \gamma_A(3^n) \Rightarrow 0.3 \leq 0.3 \]
Definition 2.5
An intuitionistic anti-fuzzy ideal $A$ of a ring $R$ is called Intuitionistic Anti Fuzzy Semiprimary Ideal (IAFSPI) if for all $a, b \in R$ either $\mu_A(ab) \geq \mu_A(a^n)$ and $\gamma_A(ab) \leq \gamma_A(a^n)$, for some $n \in \mathbb{Z}^+$ or else $\mu_A(b^m) \geq \mu_A(b^n)$ and $\gamma_A(ab) \leq \gamma_A(b^n)$ for some $m \in \mathbb{Z}^+$.

Definition 2.6 [2]
An interval-valued fuzzy set $A$ is specified by a function $M_A : E \rightarrow D[0,1]$, where $D[0,1]$ is the set of all intervals within $[0,1]$ for all $x \in E, M_A(x)$ is an interval $[a,b]$, where $0 \leq a \leq b \leq 1$.

Definition 2.7 [2]
An interval-valued intuitionistic fuzzy set $A$ over $E$ is defined as an object of the form $A = \{ x, M_A(x), N_A(x) \mid x \in E \}$, where $M_A(x) \subseteq [0,1]$ and $N_A(x) \subseteq [0,1]$ are interval and for all $x \in E$, $rmaxM_A(x) + rmaxN_A(x) \leq 1$.

Definition 2.8
Let $E_1$ and $E_2$ be two universes and let $A = \{ x, \mu_A(x), \gamma_A(x) \mid x \in E_1 \}$, $B = \{ y, \mu_B(y), \gamma_B(y) \mid y \in E_2 \}$ be two interval-valued intuitionistic fuzzy subsets of $E_1$ and $E_2$ respectively, then

$A \times B = \{(x,y), \min(rminM_A(x), rmin M_B(y)), \max(rmin N_A(x), rmin N_B(y)) \mid x \in E_1 and y \in E_2 \}$

Definition 2.9
Let $A$ be an interval-valued intuitionistic fuzzy sets. A fuzzy ideal $A$ of a ring $R$ is said to be interval-valued intuitionistic anti fuzzy primary ideal (IVIAFPI) of $R$ if for all $a, b \in R$ then either $\mu_A(ab) = rmax M_A(ab) = rmax M_A(a)$ = $\mu_A(a)$ and $\gamma_A(ab) = rmax N_A(ab)$ = $\gamma_A(a)$ or $\mu_A(a^n) = rmax M_A(a^n)$ and $\gamma_A(a^n) = rmax N_A(a^n)$, for some $n \in \mathbb{Z}^+$.

Definition 2.10
Let $A$ be an interval-valued intuitionistic fuzzy set $A$ of a ring $R$ is said to be Interval-Valued Intuitionistic Anti Fuzzy Primary Ideal (IVIAFPI) of $R$ if for all $a, b \in R$ then either $\mu_A(ab) = rmax M_A(ab) \geq rmax M_A(a^n)$ and $\gamma_A(ab) = rmax N_A(ab)$ holds for some $n \in \mathbb{Z}^+$ or $\mu_A(b^m) = rmax M_A(b^m)$ and $\gamma_A(ab) = rmax N_A(ab) \leq rmax N_A(b^n)$, for some $m \in \mathbb{Z}^+$.

Definition 2.11 [18]
A fuzzy set $\mu : L \rightarrow [0,1]$ is called a fuzzy Lie subalgebra of $L$ if

(i) $\mu(x+y) \geq \max \left( \mu(x), \mu(y) \right)$

(ii) $\mu(ax) \geq \mu(x)$

(iii) $\mu([xy]) \geq \min \left( \mu(x), \mu(y) \right)$ holds for all $x, y \in L$ and $a \in F$.

Definition 2.12 [7]
A fuzzy set $\mu : L \rightarrow [0,1]$ is called anti fuzzy Lie ideal of $L$ if

(i) $\mu(x+y) \leq \max \left( \mu(x), \mu(y) \right)$

(ii) $\mu(ax) \leq \mu(x)$

(iii) $\mu([xy]) \leq \mu(x)$ holds for all $x, y \in L$ and $a \in F$. 

3. Interval-valued Intuitionistic Anti Fuzzy Lie Primary Ideal

**Definition 3.1**

An IVIFS $A = (\mu_A, \gamma_A)$ in $L$ is called an Interval-Valued Intuitionistic Anti Fuzzy Lie Ideal (IVIAFLI) of $L$, if the following conditions are satisfied.

(i) $\overline{\mu}_A(x + y) \leq \max(\overline{\mu}_A(x), \overline{\mu}_A(y))$ and $\overline{\gamma}_A(x + y) \leq \min(\overline{\gamma}_A(x), \overline{\gamma}_A(y))$

(ii) $\overline{\mu}_A(ax) \leq \overline{\mu}_A(x)$ and $\overline{\gamma}_A(ax) \geq \overline{\gamma}_A(x)$

(iii) $\overline{\mu}_A([x + y]) \leq \overline{\mu}_A(x)$ and $\overline{\gamma}_A([x + y]) \leq \overline{\gamma}_A(x)$ for all $x, y \in L$ and $a \in F$.

**Definition 3.2**

Let $A$ be an interval-valued intuitionistic anti fuzzy Lie ideal of a Lie algebra $L$, then $A$ is said to be an Interval-Valued Intuitionistic Anti Fuzzy Lie Primary Ideal (IVIAFLPI) of $L$ if for all $x, y \in L$ and for some $n \in Z^+$, either $\overline{\mu}_A(xy) = \overline{\mu}_A(x)$ and $\overline{\gamma}_A(xy) = \overline{\gamma}_A(x)$ or $\overline{\mu}_A(xy) \geq \overline{\mu}_A(x^n)$ and $\overline{\gamma}_A(xy) \leq \overline{\gamma}_A(x^n)$ for some $n \in Z^+$.

**Definition 3.3**

Let $A$ be an IVIAFLPI of a Lie algebra $L$ then $A$ is said to be Interval-Valued Intuitionistic Anti Fuzzy Lie Semiprimary Ideal (IVIAFLSPI) of $L$ if for all $x, y \in L$ and for some $n \in Z^+$. Either $\overline{\mu}_A(xy) \geq \overline{\mu}_A(x^n)$ and $\overline{\gamma}_A(xy) \leq \overline{\gamma}_A(x^n)$ or else $\overline{\mu}_A(xy) \geq \overline{\mu}_A(y^m)$ and $\overline{\gamma}_A(xy) \leq \overline{\gamma}_A(y^m)$ for some $m \in Z^+$.

**Theorem 3.1** [13] [17]

If $A = (\overline{\mu}_A, \overline{\gamma}_A)$ is an IVIAFLPI of a Lie algebra $L$, then the anti level subset $U(A) = \{a \in L | \overline{\mu}_A(x) \leq a \}$ and $L(A, a) = \{x \in L | \overline{\gamma}_A(x) \geq a \}$ are primary ideals of $L$ for every $\overline{\gamma}_A \in I_m(\overline{\mu}_A) \cap I_m(\overline{\gamma}_A) \subseteq D[0,1]$, where $I_m(\overline{\mu}_A)$ and $I_m(\overline{\gamma}_A)$ are sets of values of $\overline{\mu}_A$ and $\overline{\gamma}_A$ respectively.

**Proof:**

Let $\overline{\gamma}_A \in I_m(\overline{\mu}_A) \cap I_m(\overline{\gamma}_A) \subseteq D[0,1]$, and let $x, y \in U(\overline{\mu}_A, \overline{\gamma}_A)$ and $a \in F$, then $\overline{\mu}_A(x) \leq I$.

where $I = [0,1]$ and $\overline{\mu}_A(x) \leq \overline{\alpha}$ it follows that $\overline{\gamma}_A(xy) = \overline{\gamma}_A(x) \leq \overline{\gamma}_A(y)$, so that $x, y \in U(\overline{\mu}_A, \overline{\gamma}_A)$, consequently $U(\overline{\mu}_A, \overline{\gamma}_A)$ IVIAFLPI of $L$. Let $x, y \in L(\overline{\gamma}_A, \overline{\gamma}_A)$ and $a \in F$, then $\overline{\gamma}_A(xy) \leq I$ where $I = [0,1]$ and $\overline{\gamma}_A(xy) \geq \overline{\gamma}_A(x)$, it follows that $\overline{\gamma}_A(xy) = \overline{\gamma}_A(x)$, so that $x, y \in L(\overline{\gamma}_A, \overline{\gamma}_A)$ consequently $L(\overline{\gamma}_A, \overline{\gamma}_A)$ is IVIAFLPI of $L$.

**Theorem 3.2** [13] [14] [16] [17]

If $A = (\overline{\mu}_A, \overline{\gamma}_A)$ and $B = (\overline{\mu}_B, \overline{\gamma}_B)$ be two IVIAFLPI of a Lie algebra $L$, then $A \times B$ is an IVIAFLPI of $L \times L$.

**Proof:**

We know that $A \times B = \{\mu_A \times \mu_B, \gamma_A \times \gamma_B \}$ where $(\overline{\mu}_A \times \overline{\gamma}_B)(x, y) = \max(\overline{\mu}_A(x), \overline{\gamma}_B(y))$ and $(\overline{\gamma}_A \times \overline{\gamma}_B)(x, y) = \min(\overline{\gamma}_A(x), \overline{\gamma}_B(y))

Let $x = (x_1, x_2)$ and $y = (y_1, y_2) \in L \times L$.

Therefore $(\overline{\mu}_A \times \overline{\gamma}_B)(xy) = (\overline{\mu}_A \times \overline{\gamma}_B)(x)$.

Now $(\overline{\gamma}_A \times \overline{\gamma}_B)(xy) = (\overline{\gamma}_A \times \overline{\gamma}_B)((x_1, x_2)(y_1, y_2)) = (\overline{\gamma}_A \times \overline{\gamma}_B)((x_1, x_2)(y_1, y_2)) = (\overline{\gamma}_A \times \overline{\gamma}_B)((x_1, x_2))(y_1, y_2)$

Therefore, $(\overline{\gamma}_A \times \overline{\gamma}_B)(xy) = (\overline{\gamma}_A \times \overline{\gamma}_B)((x_1, x_2)(y_1, y_2)) = (\overline{\gamma}_A \times \overline{\gamma}_B)((x_1, x_2))(y_1, y_2)$

and hence $A \times B$ is an IVIAFLPI of $L$. 

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Theorem 3.3 \[13]\ [14]\ [16]\ [18]

If \( A = (\overline{\mu}_A, \overline{\nu}_A) \) and \( B = (\overline{\mu}_B, \overline{\nu}_B) \) are IVIAFLPI on \( L \), then \([A, B]\) is also an IVIAFLPI of \( L \).

Proof:

Let \( A \) be an IVIAFLPI of a Lie algebra \( L \) then

\[
\langle \langle \overline{\mu}_A \rangle \rangle = \min \left( \max \left( \overline{\mu}_A (x), \overline{\mu}_B (x) \right) \right),
\]

for some \( x, y \in L \).

Consider \( x, y \in L \).

Now

\[
\langle \langle \overline{\mu}_A, \overline{\mu}_B \rangle \rangle (xy) = \min \left( \max \left( \overline{\mu}_A (x), \overline{\mu}_B (x) \right) \right)
\]

\[
/xy, x, y \in L, [xy, xy] = xy
\]

Therefore,

\[
\langle \langle \overline{\mu}_A, \overline{\mu}_B \rangle \rangle (xy) = \langle \langle \overline{\mu}_A \rangle \rangle (x)
\]

Now

\[
\langle \langle \overline{\nu}_A, \overline{\nu}_B \rangle \rangle (xy) = \max \left( \min \left( \overline{\nu}_A (x), \overline{\nu}_B (x) \right) \right)
\]

\[
/xy, x, y \in L, [xy, xy] = xy
\]

Therefore,

\[
\langle \langle \overline{\nu}_A, \overline{\nu}_B \rangle \rangle (xy) = \langle \langle \overline{\nu}_A \rangle \rangle (x)
\]

Theorem 3.4 \[13]\]

If \( A_1, A_2, B_1, B_2 \) be IVIAFLPI in \( L \) such that \( A_1 \supseteq A_2 \) and \( B_1 \supseteq B_2 \), then \([A_1, B_1] \supseteq [A_2, B_2]\).

Proof:

Consider \( x, y \in L \).

Now,

\[
\langle \langle \overline{\mu}_{A_1}, \overline{\mu}_{B_1} \rangle \rangle (xy) = \min \left( \max \left( \overline{\mu}_{A_1} (xy), \overline{\mu}_{B_1} (xy) \right) \right)
\]

\[
/xy, x, y \in L, [xy, xy] = xy
\]

\[
= \min \left( \max \left( \overline{\mu}_{A_1} (xy), \overline{\mu}_{B_1} (xy) \right) \right)
\]

\[
\leq \min \left( \max \left( \overline{\mu}_{A_2} (xy), \overline{\mu}_{B_2} (xy) \right) \right)
\]

\[
/xy, x, y \in L, [xy, xy] = xy
\]

\[
= \min \left( \max \left( \overline{\mu}_{A_2} (x), \overline{\mu}_{B_2} (x) \right) \right)
\]

\[
= \langle \langle \overline{\mu}_{A_2}, \overline{\mu}_{B_2} \rangle \rangle (x)
\]

Therefore,

\[
\langle \langle \overline{\mu}_{A_1}, \overline{\mu}_{B_1} \rangle \rangle (xy) = \langle \langle \overline{\mu}_{A_2}, \overline{\mu}_{B_2} \rangle \rangle (x)
\]

Now

\[
\langle \langle \overline{\nu}_{A_1}, \overline{\nu}_{B_1} \rangle \rangle (xy) = \max \left( \min \left( \overline{\nu}_{A_1} (xy), \overline{\nu}_{B_1} (xy) \right) \right)
\]

\[
/xy, x, y \in L, [xy, xy] = xy
\]

\[
= \max \left( \min \left( \overline{\nu}_{A_2} (xy), \overline{\nu}_{B_2} (xy) \right) \right)
\]

\[
/xy, x, y \in L, [xy, xy] = xy
\]

Therefore,

\[
\langle \langle \overline{\nu}_{A_1}, \overline{\nu}_{B_1} \rangle \rangle (xy) = \langle \langle \overline{\nu}_{A_2}, \overline{\nu}_{B_2} \rangle \rangle (x)
\]

Hence, \([A_1, B_1] \supseteq [A_2, B_2]\).

4. References


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