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Fuzzy Retraction of Fuzzy Space Time

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Abstract

Our aim in the present article is to introduce and study a new type of metric, namely fuzzy space time metric. The geodesics of fuzzy space time metric will be obtained from the view point of Lagrangian equations. Types of the fuzzy retraction of fuzzy space time metric are presented. Types of the fuzzy folding of fuzzy space time metric are defined and discussed. The deformation retraction is also discussed. Some applications concerning these relations are presented.

Keywords: Fuzzy Space Time Metric, Fuzzy Retraction, Fuzzy Deformation Retraction, Fuzzy Geodesics, Fuzzy Folding.

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1. Introduction and Definitions

As is well known, the theory of retraction is always one of interesting topics in Euclidian and Non-Euclidian space and it has been investigated from the various viewpoints by many branches of topology and differential geometry [1–27, 28, 30–33].

An n-dimensional topological manifold M is a Hausdorff topological space with a countable basis for the topology which is locally homeomorphic to R^n. If h: U U' is a homeomorphism of U M onto $U' \subseteq R^n$, then h is called a chart of M and U is the associated chart domain. A collection (h_α, u_α) is said to be an atlas for M if $\bigcup_{\alpha\in A} U_\alpha = M$ Given two charts h_α, h_β such that $U_{\alpha\beta} = U_\alpha \bigcap U_\beta \neq \varnothing$, the transformation chart $h_\beta \infty h_\alpha^{-1}$ between open sets of R^n is defined, and if all of these charts transformation are c^∞-mappings, then the manifolds under consideration is a c^∞-manifolds. A differentiable structure on M is a differentiable atlas and a differentiable manifolds is a topological manifolds with a differentiable structure [28, 30, 31, 32, 33]. M may have another structures as colour, density or any physical structures. The number of structures may be infinite. In this case the manifold is said to be a chaotic manifold [17, 23] and may become relevant to vacuum fluctuation and chaotic quantum field theories [13, 18]. The magnetic field of a magnet bar is a kind of chaotic 1-dimensional manifold represented by the magnetic flux lines. The geometric manifold is the magnetic bar itself.

Fuzzy manifolds are special type of the category of chaotic manifolds. Usually we denote by M = $M_{012\ldots h}$ to a chaotic manifolds [1, 2, 3, 6, 7, 19], where M_{0h} is the geometric (essential) manifold and the associated pure chaotic manifolds, the manifolds with physical characters, are denoted by $M_{1h}, \ldots, M_{\infty h}$.

There are many diverse applications of certain phenomena for which it is impossible to get relevant data. It may not be possible to measure essential parameters of a process such as the temperature inside molten glass or the homogeneity of a mixture inside some tanks. The required measurement scale may not exist at all, such as in the case of evaluation of offensive smells, evaluating the taste of foods or medical diagnoses by touching [1, 2, 3, 6, 9, 10, 11, 12, 18, 21, 22, 23, 2]. The aim of the present paper is to describe the above phenomena geometrically, specifically concerned with the study of the new types of fuzzy retractions, fuzzy deformation retracts and fuzzy folding of fuzzy

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space time $\tilde{M}^4$. A fuzzy manifold is manifold which has a physical character. This character is represented by the density function μ, where, $\mu \in [0, 1]$ [7, 8].

A fuzzy subset $(\tilde{A}, \mu)$ of a fuzzy manifold $(\tilde{M}, \mu)$ is called a fuzzy retraction of $(\tilde{M}, \mu)$ if there exist a continuous map $\tilde{r}: (\tilde{M}, \mu) \to (\tilde{A}, \mu)$ such that $\tilde{r}(a, \mu(a)) = (a, \mu(a))$, $\forall\, a \in \tilde{A}$, $\mu \in [0,1]$ [1, 2, 3, 4, 5, 6, 7, 21].

A fuzzy subset $(\bar{\tilde{M}}, \tilde{\mu})$ of a fuzzy manifold $(\tilde{M}, \mu)$ is called a fuzzy deformation retract if there exists a fuzzy retraction $\tilde{r}: (\tilde{M}, \mu) \to (\bar{\tilde{M}}, \tilde{\mu})$ and a fuzzy homotopy $\tilde{F}: (\tilde{M}, \mu) \times I \to (\tilde{M}, \mu)$ [2, 6, 9, 11, 22] such that:

$$\left.\begin{aligned} \tilde{F}((x, \mu), 0) &= (x, \mu) \\ \tilde{F}((x, \mu), 1) &= \tilde{r}(x, \mu) \end{aligned}\right\} x \in \tilde{M}$$

$\tilde{F}((a, \mu), t) = (a, \mu)$, $\forall\, (a, \mu) \in \bar{\tilde{M}}$, $t \in I$, $\mu \in [0,1]$. where $\tilde{r}(x, \mu)$ is the retraction mentioned above. Topological folding of $\tilde{M}^4$ [2, 3].

The Space Time metric. The line element representing this space time is given by [25, 29].

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \left(1 - \frac{2M}{r}\right)^{-1} dr^2 - \left(1 - \frac{\alpha}{r}\right) r^2 \left(d\theta^2 + \sin^2 \theta\, d\phi^2\right),$$

where $\alpha = Q^2 \frac{\exp(-2\Phi_0)}{M}$, m and Q are, respectively mass and charged parameters; Φ_0 is the asymptotic value of dilation field.

A map $\tilde{\Im}: \tilde{M}^4 \to \tilde{M}^4$, is said to be an isometric folding of fuzzy Space Time $\tilde{M}^4$ into itself iff for any piecewise fuzzy geodesic path $\gamma: J \to \tilde{M}^4$ the induced path $\tilde{\Im} \circ \gamma: J \to \tilde{M}^4$ is a piecewise fuzzy geodesic and of the same length as γ, where J = [0,1]. If $\tilde{\Im}$ does not preserve lengths, then $\tilde{\Im}$ is a topological folding of fuzzy Space Time $\tilde{M}^4$ [1, 4, 5, 6, 8, 9, 22]. The isofuzzy folding of $\cup \tilde{M}_i$ is a folding $\tilde{f}: \cup \tilde{M}_i \to \cup \tilde{M}_i$ such that $\tilde{f}(\tilde{M}) = \tilde{M}$ and any $\tilde{M}_i$ belong to the upper hypermanifolds $\exists \tilde{M}_j$ down $\tilde{M}$ such that $\mu_i = \mu_j$ for every corresponding points i.e. $\mu(a_i) = \mu(a_j)$ [6]. See Figure (1).

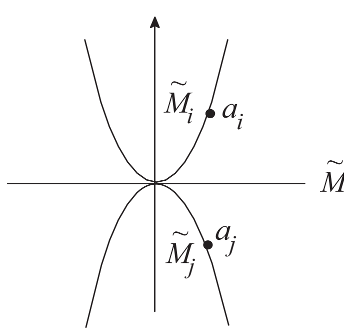

Figure 1. The isofuzzy folding of U ~*Mi* [23].

2. Main Results

In what follows, we would like to evaluate the geodesic of fuzzy space time $\tilde{M}^4$. The fuzzy metric is defined as:

$$ds^2 = \left(1 - \frac{2m}{r(\eta)}\right) dt^2(\eta) - \left(1 - \frac{2m}{r(\eta)}\right)^{-1} dr^2(\eta) - \left(1 - \frac{\alpha}{r(\eta)}\right) r^2(\eta) \left(d\theta^2(\eta) + \sin^2 \theta(\eta)\, d\phi^2(\eta)\right), \quad (1)$$

where, $r(\eta)$, $t(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ are functions of energy distribution.

Then the coordinates of the fuzzy space time $\tilde{M}^4$ are

$$\begin{aligned} \tilde{x}_1^2 &= c_1 + \left(1 - \frac{2m}{r(\eta)}\right) t^2(\eta), \\ \tilde{x}_2^2 &= c_2 - \left(r^2(\eta) + 4mr(\eta)\right) + 3m^2 \ln\left(r(\eta) - 2m\right), \\ \tilde{x}_3^2 &= c_3 - \left(1 - \frac{\alpha}{r(\eta)}\right) r^2(\eta)\, \theta^2(\eta), \\ \tilde{x}_4^2 &= c_4 - \left(1 - \frac{\alpha}{r(\eta)}\right) r^2(\eta) \sin^2 \theta(\eta)\, \phi^2(\eta). \end{aligned} \quad (2)$$

where, c_1, c_2, c_3, and c_4, are the constant of integration. In a position, using lagrangian equations

$$\frac{d}{ds}\left(\frac{\partial T}{\partial \psi_i'}\right) - \left(\frac{\partial T}{\partial \psi_i}\right) = 0, \; i = 1, 2, 3, 4, 5.$$

To deduce a fuzzy retraction which is fuzzy geodesic in fuzzy space time $\tilde{M}^4$. Since $T = \frac{1}{2} ds^2$, this yields

$$T = \frac{1}{2}\left[\begin{aligned} &\left(1 - \frac{2m}{r(\eta)}\right) t'^2(\eta) - \left(1 - \frac{2m}{r(\eta)}\right)^{-1} r'^2(\eta) - \\ &\left(1 - \frac{\alpha}{r(\eta)}\right) r^2(\eta) \left(\theta'^2(\eta) + \sin^2 \theta(\eta)\, \phi'^2(\eta)\right) \end{aligned}\right], \quad (3)$$

Then the Lagrangian equations are

$$\frac{d}{ds}\left(\left(1-\frac{2m}{r(\eta)}\right)t'^2(\eta)\right)=0, \tag{4}$$

$$\frac{d}{ds}\left(\left(1-\frac{2m}{r(\eta)}\right)t'^2(\eta)\right)+\frac{m}{r^2(\eta)}t'^2(\eta)+\frac{m}{(r(\eta)-2m)^2}r'^2(\eta)-\left(r(\eta)-\frac{\alpha}{2}\right)\left(\theta^2(\eta)+\sin^2\theta(\eta)\phi^2(\eta)\right)=0, \tag{5}$$

$$\frac{d}{ds}\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta'(\eta)-\left(\left(1-\frac{\alpha}{r(\eta)}\right)\right)r^2(\eta)\sin\theta(\eta)\cos\theta(\eta)\,\phi^2(\eta)=0, \tag{6}$$

$$\frac{d}{ds}\left(\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi'(\eta)\right)=0, \tag{7}$$

$$\frac{d}{ds}\left(\left(1-\frac{2m}{r(\eta)}\right)t'(\eta)t''(\eta)-\left(1-\frac{2m}{r(\eta)}\right)^{-1}r'(\eta)r''(\eta)-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\begin{pmatrix}\theta(\eta)\theta'(\eta)+\sin^2\theta(\eta)\phi(\eta)\phi'(\eta)+\frac{r'(\eta)m}{r^2(\eta)}t'^2(\eta)\\+\frac{r'(\eta)m}{(r(\eta)-2m)^2}r'^2(\eta)-\left(r(\eta)-\frac{\alpha}{2}\right)r'(\eta)\left(\theta^2(\eta)\right.\\\left.+\sin^2\theta(\eta)\phi(\eta)\phi^2(\eta)\right)\end{pmatrix}\right)=0, \tag{8}$$

from equation (7) we have $\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi'(\eta)$ = constant say β, β = 0, we get φ′ (η) = 0, which implies to φ(η) = 0 . Then, we obtain the following coordinates

$$\tilde{x}_1^2=c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta),$$

$$\tilde{x}_2^2=c_2-\left(r^2(\eta)+4mr(\eta)+3m^2\ln(r(\eta)-2m\right),$$

$$\tilde{x}_3^2=c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta(\eta),\tilde{x}_4^2=c_4.$$

which is the fuzzy hyperaffine subspace geodesic $\tilde{M}_1$ in fuzzy space time $\tilde{M}^4$ which is the fuzzy retraction.

Also, if $\sin^2\theta(\eta)=0$, we obtain the fuzzy affine subspace $\tilde{M}_2$ in fuzzy space time $\tilde{M}^4$ which represented by

$$\tilde{x}_1^2=c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta),$$

$$\tilde{x}_2^2=c_2-\left(r^2(\eta)+4mr(\eta)\right)+3m^2\ln\left(r(\eta)-2m\right),$$

$$\tilde{x}_3^2=c_3,\tilde{x}_4^2=c_4.$$

which is affine subspace geodesic in fuzzy space time $\tilde{M}^4$. Also, this fuzzy geodesic is a fuzzy retraction.

If $r^2(\eta)=0$, we have the following coordinates

$$\tilde{x}_1^2=c_1,\tilde{x}_2^2=c_2,\tilde{x}_3^2=c_3,\ \tilde{x}_4^2=c_4.$$

Then, $\tilde{x}_1^2+\tilde{x}_2^2+\tilde{x}_3^2-\tilde{x}_4^2=c_1^2+c_2^2+c_3^2-c_4^2$. Now, we see that if $\left(c_1^2+c_2^2+c_3^2\right)\succ c_4^2$ corresponding to fuzzy space like hypersphere $\tilde{S}_1^3$ geodesic. If $\left(c_1^2+c_2^2+c_3^2\right)\prec c_4^2$ we obtain fuzzy time-like geodesic hypersphere $\tilde{S}_2^3$. Also, if $\left(c_1^2+c_2^2+c_3^2\right)=c_4^2$ corresponding to a null geodesic hypersphere $\tilde{S}_3^3$. If $\left(1-\frac{\alpha}{r(\eta)}\right)$=0, the fuzzy affine subspace of fuzzy space time $\tilde{M}_3$ is represented by the same coordinates as $\tilde{M}_2$.

Then, the following theorem has been proved.

Theorem 1:
The geodesic retractions of fuzzy space time $\tilde{M}^4$ are fuzzy hypersurface, fuzzy hypersphere and fuzzy curves.

In this position, we present some cases of fuzzy deformation retract of fuzzy open space time $\tilde{M}^4$. The fuzzy deformation retract of fuzzy open space time $\tilde{M}^4$ is defined by

$$\tilde{\varphi}:\left(\tilde{M}^4-\{\tilde{p}_i\}\right)\times I\to\left(\tilde{M}^4-\{\tilde{p}_i\}\right),\ \text{where}\ \left(\tilde{M}^4-\{\tilde{p}_i\}\right)$$

be the fuzzy open space time $\tilde{M}^4$ and I is the closed interval [0,1], be present as

$$\tilde{\varphi}:\left\{\left[\begin{pmatrix}\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\sqrt{\begin{pmatrix}c_2-\left(r^2(\eta)+4mr(\eta)\right)\\+3m^2\ln\left(r(\eta)-2m\right)\end{pmatrix}},\\\sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta(\eta)\right)},\\\sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi(\eta)\right)}-\{\tilde{p}_i\}\end{pmatrix}\right]\right\}\times I\to$$

$$\left\{\left[\begin{pmatrix}\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\sqrt{\begin{pmatrix}c_2-\left(r^2(\eta)+4mr(\eta)\right)\\+3m^2\ln\left(r(\eta)-2m\right)\end{pmatrix}},\\\sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta(\eta)\right)},\\\sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi(\eta)\right)}-\{\tilde{p}_i\}\end{pmatrix}\right]\right\}$$

The fuzzy deformation retract of the fuzzy space time $\tilde{M}^4$ into fuzzy geodesic $\tilde{M}_1 \subset \tilde{M}^4$ is given by

$$\tilde{\varphi}(x,\nu)=\sqrt{(1-\nu)}\left\{\begin{bmatrix}\left(\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\right.\\ \sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)\right)+3m^2\ln\left(r(\eta)-2m\right)\right)},\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta(\eta)\right)},\\ \left.\sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi(\eta)\right)}-\{\tilde{p}_i\}\right)\end{bmatrix}\right\}+$$

$$\nu\left\{\begin{bmatrix}\left(\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\right.\\ \sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)+3m^2\ln\left(r(\eta)-2m\right)\right)\right)},\\ \left.\sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta(\eta)\right)},\sqrt{c_4}\right)\end{bmatrix}\right\}$$

where

$$\tilde{\varphi}(x,0)=\left\{\begin{bmatrix}\left(\sqrt{\left(c_1+(1-\frac{2m}{r(\eta)})t^2(\eta)\right)},\right.\\ \sqrt{\left(c_2-(r^2(\eta)+4mr(\eta)+3m^2\ln(r(\eta)-2m)\right)},\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta(\eta)\right)},\\ \left.\sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi(\eta)\right)}-\{\tilde{p}_i\}\right)\end{bmatrix}\right\}$$

and

$$\tilde{\varphi}(x,1)=\left\{\begin{bmatrix}\left(\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\right.\\ \sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)\right)+3m^2\ln\left(r(\eta)-2m\right)\right)},\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta(\eta)\right)},\\ \left.\sqrt{c_4}\right)\end{bmatrix}\right\}$$

The fuzzy deformation retract of the fuzzy space time $\tilde{M}^4$ into geodesic $\tilde{M}_2 \subset \tilde{M}^4$ is given by

$$\tilde{\varphi}(x,\nu)=$$

$$\cos\frac{\pi\nu}{2}\left\{\begin{bmatrix}\left(\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\right.\\ \sqrt{\begin{matrix}(c_2-r^2(\eta)+4mr(\eta))+\\ 3m^2\ln\left(r(\eta)-2m\right),\end{matrix}}\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta(\eta)\right)},\\ \sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi(\eta)\right)}\\ \left.-\{\tilde{p}_i\}\right)\end{bmatrix}\right\}$$

$$+\tan\frac{\pi\nu}{2}\left\{\begin{bmatrix}\left(\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\right.\\ \sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)+3m^2\ln\left(r(\eta)-2m\right)\right)\right)},\\ \left.\sqrt{c_3},\sqrt{c_4}\right)\end{bmatrix}\right\}.$$

The fuzzy deformation retracts of the fuzzy space time $\tilde{M}^4$ into geodesic $\tilde{S}_1^3 \subset \tilde{M}^4$ is given by

$$\tilde{\varphi}(x,\nu)=(1-\nu)\left\{\begin{bmatrix}\left(\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\right.\\ \sqrt{\begin{matrix}\left(c_2-\left(r^2(\eta)+4mr(\eta)\right)+\right.\\ 3m^2\ln\left(r(\eta)-2m\right),\end{matrix}}\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta(\eta)\right)},\\ \sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi(\eta)\right)}\\ \left.-\{\tilde{p}_i\}\right)\end{bmatrix}\right\}$$

$$+\sin\frac{\pi\nu}{2}\left\{\left(\sqrt{c_1},\sqrt{c_2},\sqrt{c_3},\sqrt{c_4}\right\}.\right.$$

Now, we are going to discuss the folding $\tilde{\Im}$ of the fuzzy space time $\tilde{M}^4$. Let $\tilde{\Im}: \tilde{M}^4 \to \tilde{M}^4$, where

$$\tilde{\Im}\left(\tilde{x}_1^2,\tilde{x}_2^2,\tilde{x}_3^2,\tilde{x}_4^2\right)=\left(\left|\tilde{x}_1^2\right|,\tilde{x}_2^2,\tilde{x}_3^2,\tilde{x}_4^2\right)$$

An isometric folding of the fuzzy space time $\tilde{M}^4$ into itself may be defined by

$$\tilde{\Im}:\left\{\left[\left(\begin{array}{l}\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\\ \sqrt{\left(\begin{array}{l}c_2-\left(r^2(\eta)+4mr(\eta)+\right.\\ \left.3m^2\ln\left(r(\eta)-2m\right)\right)\end{array}\right)},\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)\right)},\\ \sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)\right)}-\{\tilde{p}_i\}\end{array}\right)\right]\right\}\to$$

$$\left\{\left[\left(\begin{array}{l}\left|\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)}\right|,\\ \sqrt{\left(\begin{array}{l}c_2-\left(r^2(\eta)+4mr(\eta)+\right.\\ \left.3m^2\ln\left(r(\eta)-2m\right)\right)\end{array}\right)},\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)\right)},\\ \sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)\right)}\\ -\{\tilde{p}_i\}\end{array}\right)\right]\right\}$$

The fuzzy deformation retracts of the folded fuzzy space time $\tilde{M}^4$ into the folded fuzzy geodesic $\tilde{\Im}\left(\tilde{M}_j\right)\subset\tilde{\Im}\left(\tilde{M}^4\right)$ is defined by

$$\tilde{\varphi}_{\tilde{\Im}}:\left(\tilde{\Im}\left(\tilde{M}^4\right)-\{\tilde{p}_i\}\right)\times I\to\left(\tilde{\Im}\left(\tilde{M}^4\right)-\{\tilde{p}_i\}\right)$$

where $\left(\tilde{\Im}\left(\tilde{M}^4\right)-\{\tilde{p}_i\}\right)$ be the folded fuzzy open space time $\tilde{M}^4$ and $\{\tilde{p}_i\}$ any two points in the fuzzy space time $\tilde{M}^4$ and also I is the closed interval [0, 1]. The folded fuzzy retraction of the folded fuzzy space time $\tilde{M}^4$ is given by

$$\tilde{\mathfrak{R}}_{\tilde{\Im}}:\left(\tilde{\Im}\left(\tilde{M}^4\right)-\{\tilde{p}_i\}\right)\to\tilde{\Im}\left(\tilde{M}_1\right),\ \tilde{\Im}\left(\tilde{M}_2\right),\ \tilde{\Im}\left(\tilde{S}_1^3\right)$$

The fuzzy deformation retracts of the folded fuzzy space time $\tilde{\Im}\left(\tilde{M}^4\right)$ into the folded fuzzy geodesic $\tilde{\Im}\left(\tilde{M}_1\right)\subset\tilde{\Im}\left(\tilde{M}^4\right)$ is given by

$$\tilde{\varphi}_{\tilde{\Im}}:\left\{\left[\left(\begin{array}{l}\left|\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)}\right|,\sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)+3m^2\ln\left(r(\eta)-2m\right)\right)\right)},\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)\right)},\\ \sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)\right)}-\{\tilde{p}_i\}\end{array}\right)\right]\right\}\times I\to$$

$$\left\{\left[\left(\begin{array}{l}\left|\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)}\right|,\sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)+3m^2\ln\left(r(\eta)-2m\right)\right)\right)},\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)\right)},\\ \sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)\right)}-\{\tilde{p}_i\}\end{array}\right)\right]\right\}$$

with

$$\tilde{\varphi}_{\tilde{\Im}}(x,v)=e(1-v)e^{(v-1)}$$

$$\left\{\left[\left(\begin{array}{l}\left|\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)}\right|,\\ \sqrt{\left(\begin{array}{l}c_2-\left(r^2(\eta)+4mr(\eta)+\right.\\ \left.3m^2\ln\left(r(\eta)-2m\right)\right)\end{array}\right)},\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)\right)},\\ \sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)\right)}-\{\tilde{p}_i\}\end{array}\right)\right]\right\}+$$

$$ve^{(1-v)}\left\{\left[\left(\begin{array}{l}\left|\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)}\right|,\\ \sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)+3m^2\ln\left(r(\eta)-2m\right)\right)\right)},\\ \sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)\right)},\sqrt{c_4}\end{array}\right.\right.\right.$$

Then, the following theorem has been proved.

Theorem 2:
Under the defined fuzzy folding and any folding homeomorphic to this type of folding, the fuzzy deformation

retracts of the folded fuzzy space time $\tilde{M}^4$ is different from the fuzzy deformation retract of fuzzy space time $\tilde{M}^4$ into the fuzzy geodesic.

Now, if the fuzzy folding is defined by $\tilde{\Im}^*:\tilde{M}^4\to\tilde{M}^4$, where

$$\tilde{\Im}^*:\left(\tilde{x}_1^2,\tilde{x}_2^2,\tilde{x}_3^2,\tilde{x}_4^2\right)=\left(\tilde{x}_1^2,\tilde{x}_2^2,\left|\tilde{x}_3^2\right|,\tilde{x}_4^2\right)$$

The isometric folded of the fuzzy space time $\tilde{\Im}^*\left(\tilde{M}^4\right)$ is defined as,

$$\tilde{W}^*=\left\{\begin{bmatrix}\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta\right)},\\ \sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)+3m^2\ln\left(r(\eta)-2m\right)\right)\right)},\\ \left|\sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)\right)}\right|,\\ \sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)\right)}-\{\tilde{p}_i\}\end{bmatrix}\right\}$$

The fuzzy deformation retract of the folded fuzzy space time $\tilde{\Im}^*\left(\tilde{M}^4\right)$ into the folded fuzzy geodesic $\tilde{\Im}^*\left(\tilde{M}_2\right)$ is given by

$$\tilde{\varphi}_{\Im}(x,\nu)=\sqrt{(1-\nu)}\left\{\begin{bmatrix}\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta)\right)},\\ \sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)+3m^2\ln\left(r(\eta)-2m\right)\right)\right)},\\ \left|\sqrt{\left(c_3-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\theta^2(\eta)\right)}\right|,\\ \sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi^2(\eta)\right)}-\{\tilde{p}_i\}\end{bmatrix}\right.$$

$$+2\sqrt{(\nu\div 4)}\left\{\begin{bmatrix}\sqrt{\left(c_1+\left(1-\frac{2m}{r(\eta)}\right)t^2(\eta\right)},\\ \sqrt{\left(c_2-\left(r^2(\eta)+4mr(\eta)+3m^2\ln\left(r(\eta)-2m\right)\right)\right)},\\ \left|\sqrt{c_3}\right|,\sqrt{\left(c_4-\left(1-\frac{\alpha}{r(\eta)}\right)r^2(\eta)\sin^2\theta(\eta)\phi(\eta)\right)}\end{bmatrix}\right\}$$

Hence, we can formulate the following theorem.

Theorem 3:
Under the defined fuzzy folding and any folding homeomorphic to this type of folding, the fuzzy deformation retracts of the folded fuzzy space time $\tilde{M}^4$ into the folded fuzzy geodesic is different from the fuzzy space time $\tilde{M}^4$ into the fuzzy geodesic.

Theorem 4:
The end of the limits of the fuzzy foldings of fuzzy space time $\tilde{M}^4$ is a 0-dimensional space.

Proof:

Let $\tilde{\Im}_1:\tilde{M}^4\to\tilde{M}^4,\tilde{\Im}_2:\tilde{\Im}_1\left(\tilde{M}^4\right)\to\tilde{\Im}_1\left(\tilde{M}^4\right),$

$$\tilde{\Im}_3:\tilde{\Im}_2\left(\tilde{\Im}_1\left(\tilde{M}^4\right)\right)\to\tilde{\Im}_2\left(\tilde{\Im}_1\left(\tilde{M}^4\right)\right),\ldots,$$

$$\tilde{\Im}_n:\tilde{\Im}_{n-1}\left(\tilde{\Im}_{n-2}\ldots\left(\tilde{\Im}_3\left(\tilde{\Im}_2\left(\tilde{\Im}_1\left(\tilde{M}^4\right)\right)\right)\right)\ldots\right)\to$$

$$\tilde{\Im}_{n-1}\left(\tilde{\Im}_{n-2}\ldots\left(\tilde{\Im}_3\left(\tilde{\Im}_2\left(\tilde{\Im}_1\left(\tilde{M}^4\right)\right)\right)\right)\ldots\right),$$

$$\lim_{n\to\infty}\tilde{\Im}_n\left(\tilde{\Im}_{n-1}\left(\tilde{\Im}_{n-2}\ldots\left(\tilde{\Im}_3\left(\tilde{\Im}_2\left(\tilde{\Im}_1\left(\tilde{M}^4\right)\right)\right)\right)\ldots\right)\right)=\tilde{M}^3.$$

Let

$$\tilde{K}_1:\tilde{M}^3\to\tilde{M}^3,\ \tilde{K}_2:\tilde{K}_1\left(\tilde{M}^3\right)\to\tilde{K}_1\left(\tilde{M}^3\right),$$

$$\tilde{K}_3:\tilde{K}_2\left(\tilde{K}_1\left(\tilde{M}^3\right)\right)\to\tilde{K}_2\left(\tilde{K}_1\left(\tilde{M}^3\right)\right),\ldots,$$

$$\tilde{K}_n:\tilde{K}_{n-1}\left(\tilde{K}_{n-2}\ldots\left(\tilde{K}_3\left(\tilde{K}_2\left(\tilde{K}_1\left(\tilde{M}^3\right)\right)\right)\right)\ldots\right)\to$$

$$\tilde{K}_{n-1}\left(\tilde{K}_{n-2}\ldots\left(\tilde{K}_3\left(\tilde{K}_2\left(\tilde{K}_1\left(\tilde{M}^3\right)\right)\right)\right)\ldots\right),$$

$$\lim_{n\to\infty}\tilde{K}_n\left(\tilde{K}_{n-1}\left(\tilde{K}_{n-2}\ldots\left(\tilde{K}_3\left(\tilde{K}_2\left(\tilde{K}_1\left(\tilde{M}^3\right)\right)\right)\right)\ldots\right)\right)$$

$$=\tilde{M}^2$$

Consequently,

$\lim\limits_{g\to\infty}\lim\limits_{s\to\infty}\lim\limits_{m\to\infty}\lim\limits_{n\to\infty}\tilde{\eta}_g\left(\tilde{h}_S\left(\tilde{K}_m\left(\tilde{\Im}_n\left(\tilde{M}^4\right)\right)\right)\right)=\tilde{M}^0=0$ -dimensional sphere, it is a minimal geodesic in fuzzy space time $\tilde{M}^4$ and also minimal retraction.

Theorem 5:
The folding of the fuzzy space time $\tilde{M}^4$, into itself induces two chains of fuzzy folding $\bar{\tilde{M}}^4$ and $\underline{\tilde{M}}^4$ which is a type of fuzzy retractions.

Proof:
Assume $\tilde{\Im}_1:\tilde{M}^4\to\tilde{M}^4$ be a fuzzy folding from $\tilde{M}^4$ into $\tilde{M}^4$ such that $\tilde{\Im}_1(\tilde{M}^4)\neq\tilde{M}^4$. This folding induces 2-chains

of fuzzy folding $\bar{\tilde{\Im}}_1 : \bar{\tilde{M}}^4 \to \bar{\tilde{M}}^4$ such that $\bar{\tilde{\Im}}_1(\bar{\tilde{M}}^4) \neq \bar{\tilde{M}}^4$ and $\underline{\tilde{\Im}}_1 : \underline{\tilde{M}}^4 \to \underline{\tilde{M}}^4$ such that $\underline{\tilde{\Im}}_1(\underline{\tilde{M}}^4) \neq \underline{\tilde{M}}^4$, where $\dim \tilde{\Im}_1 = \dim \bar{\tilde{\Im}}_1 = \dim \underline{\tilde{\Im}}_1 \neq \dim \tilde{R}_1$. Then the fuzzy folding not conside with the fuzzy retraction. Also, let $\tilde{\Im}_2 : \tilde{\Im}_1\left(\tilde{M}^4\right) \to \tilde{\Im}_1\left(\tilde{M}^4\right)$, this folding induces $\bar{\tilde{\Im}}_2$ and $\underline{\tilde{\Im}}_2$ where

$$\bar{\tilde{\Im}}_2 : \bar{\tilde{\Im}}_1\left(\bar{\tilde{M}}^4\right) \to \bar{\tilde{\Im}}_1\left(\bar{\tilde{M}}^4\right),$$

$$\underline{\tilde{\Im}}_2 : \underline{\tilde{\Im}}_1\left(\underline{\tilde{M}}^4\right) \to \underline{\tilde{\Im}}_1\left(\underline{\tilde{M}}^4\right),$$

$$\tilde{\Im}_3 : \tilde{\Im}_2\left(\tilde{\Im}_1\left(\tilde{M}^4\right)\right) \to \tilde{\Im}_2\left(\tilde{\Im}_1\left(\tilde{M}^4\right)\right)$$

Where $\bar{\tilde{\Im}}_3 : \bar{\tilde{\Im}}_2\left(\bar{\tilde{\Im}}_1\left(\bar{\tilde{M}}^4\right)\right) \to \bar{\tilde{\Im}}_2\left(\bar{\tilde{\Im}}_1\left(\bar{\tilde{M}}^4\right)\right)$,

$$\underline{\tilde{\Im}}_3 : \underline{\tilde{\Im}}_2\left(\underline{\tilde{\Im}}_1\left(\underline{\tilde{M}}^4\right)\right) \to \underline{\tilde{\Im}}_2\left(\underline{\tilde{\Im}}_1\left(\underline{\tilde{M}}^4\right)\right), \ \ldots\ ,$$

$$\tilde{\Im}_n : \tilde{\Im}_{n-1}\left(\tilde{\Im}_{n-2}\left(\tilde{\Im}_{n-3} \ldots \tilde{\Im}_2\left(\tilde{\Im}_1\left(\tilde{M}^4\right)\right)\ldots\right)\right)$$

$$\to \tilde{\Im}_{n-1}\left(\tilde{\Im}_{n-2}\left(\tilde{\Im}_{n-3} \ldots \tilde{\Im}_2\left(\tilde{\Im}_1\left(\tilde{M}^4\right)\right)\ldots\right)\right).$$

Then the fuzzy folding induces two chains of fuzzy foldings,

$$\bar{\tilde{\Im}}_n : \bar{\tilde{\Im}}_{n-1}\left(\bar{\tilde{\Im}}_{n-2}\left(\bar{\tilde{\Im}}_{n-3} \ldots \bar{\tilde{\Im}}_2\left(\bar{\tilde{\Im}}_1\left(\bar{\tilde{M}}^4\right)\right)\ldots\right)\right) \to$$

$$\bar{\tilde{\Im}}_{n-1}\left(\bar{\tilde{\Im}}_{n-2}\left(\bar{\tilde{\Im}}_{n-3} \ldots \bar{\tilde{\Im}}_2\left(\bar{\tilde{\Im}}_1\left(\bar{\tilde{M}}^4\right)\right)\ldots\right)\right),$$

$$\underline{\tilde{\Im}}_n : \underline{\tilde{\Im}}_{n-1}\left(\underline{\tilde{\Im}}_{n-2}\left(\underline{\tilde{\Im}}_{n-3} \ldots \underline{\tilde{\Im}}_2\left(\underline{\tilde{\Im}}_1\left(\underline{\tilde{M}}^4\right)\right)\ldots\right)\right) \to$$

$$\underline{\tilde{\Im}}_{n-1}\left(\underline{\tilde{\Im}}_{n-2}\left(\underline{\tilde{\Im}}_{n-3} \ldots \underline{\tilde{\Im}}_2\left(\underline{\tilde{\Im}}_1\left(\underline{\tilde{M}}^4\right)\right)\ldots\right)\right).$$

Corollary 1:

The fuzzy retraction of the fuzzy space time $\tilde{M}^4$, induces two chains of fuzzy retractions of the two fuzzy systems of fuzzy manifolds homeomorphic to $\cup \bar{\tilde{M}}^4$ and $\cup \underline{\tilde{M}}^4$.

Theorem 6:

If the fuzzy retraction of $\tilde{M}^4$, is $\tilde{r} : \left(\tilde{M}^4 - \tilde{\beta}_1\right) \to \tilde{M}_2$, and the fuzzy folding of $\left(\tilde{M}^4 - \tilde{\beta}_1\right)$ into itself is $\tilde{\Im} : \left(\tilde{M}^4 - \tilde{\beta}_1\right) \to \left(\tilde{M}^4 - \tilde{\beta}_1\right)$, then there are induces two chains of fuzzy retractions and foldings such that the following diagrams are commutative.

Proof:

Let the fuzzy retraction of $\tilde{M}^4$ is defined by $\tilde{r}_1 : \left(\tilde{M}^4 - \tilde{\beta}_1\right) \to \tilde{M}_2$, and the fuzzy folding of $\tilde{M}^4, \tilde{M}_2$, are given by $\tilde{\Im}_1\left(\tilde{M}^4 - \tilde{\beta}_1\right) \to \left(\tilde{M}^4 - \tilde{\beta}_1\right)$, $\tilde{\Im}_2 : \tilde{M}_2 \to \tilde{M}_2$. Also, $\tilde{r}_2 : \tilde{\Im}_1\left(\tilde{M}^4 - \tilde{\beta}_1\right) \to \tilde{M}_2$. Then there are induced two chains of fuzzy retractions and foldings are given by

$$\bar{\tilde{r}}_1 : \left(\bar{\tilde{M}}^4 - \bar{\tilde{\beta}}_1\right) \to \bar{\tilde{M}}_2, \quad \underline{\tilde{r}}_1 : \left(\underline{\tilde{M}}^4 - \underline{\tilde{\beta}}_1\right) \to \underline{\tilde{M}}_2$$

$$\bar{\tilde{\Im}}_1 : \left(\bar{\tilde{M}}^4 - \bar{\tilde{\beta}}_1\right) \to \left(\bar{\tilde{M}}^4 - \bar{\tilde{\beta}}_1\right), \ \underline{\tilde{\Im}}_1 : \left(\underline{\tilde{M}}^4 - \underline{\tilde{\beta}}_1\right) \to \left(\underline{\tilde{M}}^4 - \underline{\tilde{\beta}}_1\right).$$

$$\bar{\tilde{r}}_2 : \bar{\tilde{\Im}}_1\left(\bar{\tilde{M}}^4 - \bar{\tilde{\beta}}_1\right) \to \bar{\tilde{M}}_2, \quad \underline{\tilde{r}}_2 : \underline{\tilde{\Im}}_1\left(\underline{\tilde{M}}^4 - \underline{\tilde{\beta}}_1\right) \to \underline{\tilde{M}}_2$$

$$\bar{\tilde{\Im}}_2 : \bar{\tilde{M}}_2 \to \bar{\tilde{M}}_2, \ \underline{\tilde{\Im}}_2 : \underline{\tilde{M}}_2 \to \underline{\tilde{M}}_2$$

Hence, the following diagrams are commutative

$$\begin{tikzcd}
\left(\tilde{M}^4 - \tilde{\beta}_1\right) \arrow[r, "\tilde{r}_1"] \arrow[d, "\tilde{\Im}_1"'] & \tilde{M}_2 \arrow[d, "\tilde{\Im}_2"] \\
\left(\tilde{M}^4 - \tilde{\beta}_1\right) \arrow[r, "\tilde{r}_2"] & \tilde{M}_2
\end{tikzcd}$$

$$\begin{tikzcd}
\left(\bar{\tilde{M}}^4 - \bar{\tilde{\beta}}_1\right) \arrow[r, "\bar{\tilde{r}}_1"] \arrow[d, "\bar{\tilde{\Im}}_1"'] & \bar{\tilde{M}}_2 \arrow[d, "\bar{\tilde{\Im}}_2"] \\
\left(\bar{\tilde{M}}^4 - \bar{\tilde{\beta}}_1\right) \arrow[r, "\bar{\tilde{r}}_2"] & \bar{\tilde{M}}_2
\end{tikzcd}
\qquad
\begin{tikzcd}
\left(\underline{\tilde{M}}^4 - \underline{\tilde{\beta}}_1\right) \arrow[r, "\underline{\tilde{r}}_1"] \arrow[d, "\underline{\tilde{\Im}}_1"'] & \underline{\tilde{M}}_2 \arrow[d, "\underline{\tilde{\Im}}_2"] \\
\left(\underline{\tilde{M}}^4 - \underline{\tilde{\beta}}_1\right) \arrow[r, "\underline{\tilde{r}}_2"] & \underline{\tilde{M}}_2
\end{tikzcd}$$

Corollary 2:

The generalization of theorem (6) represented by the following chins

$$\tilde{r}_{i+1} \circ \tilde{\Im}_i = \tilde{\Im}_{i+1} \circ \tilde{r}_i,$$

$$\bar{\tilde{r}}_{i+1} \circ \bar{\tilde{\Im}}_i = \bar{\tilde{\Im}}_{i+1} \circ \bar{\tilde{r}}_i, \ \underline{\tilde{r}}_{i+1} \circ \underline{\tilde{\Im}}_i = \underline{\tilde{\Im}}_{i+1} \circ \underline{\tilde{r}}_i,$$

i=1,2,…,n.

Corollary 3:

The relation between the fuzzy retraction and the limit of the fuzzy folding discussed from the following commutative diagram

$$\begin{array}{ccc}
\left(\tilde{M}^4-\tilde{\beta}_1\right) & \xrightarrow{\tilde{r}_1} & \left(\tilde{M}_1-\tilde{\beta}_2\right)\\
\lim\limits_{m\to\infty}\tilde{\Im}_m\downarrow & & \downarrow \lim\limits_{m\to\infty}\tilde{\Im}_{m+1}\\
\left(\tilde{M}_1-\tilde{\beta}_2\right) & \xrightarrow{\tilde{r}_2} & \left(\tilde{M}_2-\tilde{\beta}_3\right)
\end{array}$$

$$\begin{array}{ccc}
\left(\bar{\tilde{M}}^4-\bar{\tilde{\beta}}_1\right) & \xrightarrow{\bar{\tilde{r}}_1} & \left(\bar{\tilde{M}}_1-\bar{\tilde{\beta}}_2\right)\\
\downarrow \lim\limits_{m\to\infty}\bar{\tilde{\Im}}_m & & \downarrow \lim\limits_{m\to\infty}\bar{\tilde{\Im}}_{m+1}\\
\left(\bar{\tilde{M}}_1-\bar{\tilde{\beta}}_2\right) & \xrightarrow[\bar{\tilde{r}}_2]{} & \left(\bar{\tilde{M}}_2-\bar{\tilde{\beta}}_3\right)
\end{array}
\qquad
\begin{array}{ccc}
\left(\underline{\tilde{M}}^4-\underline{\tilde{\beta}}_1\right) & \xrightarrow{\underline{\tilde{r}}_1} & \left(\underline{\tilde{M}}_1-\underline{\tilde{\beta}}_2\right)\\
\downarrow \lim\limits_{m\to\infty}\underline{\tilde{\Im}}_m & & \downarrow \lim\limits_{m\to\infty}\underline{\tilde{\Im}}_{m+1}\\
\left(\underline{\tilde{M}}^4-\underline{\tilde{\beta}}_1\right) & \xrightarrow[\underline{\tilde{r}}_2]{} & \left(\underline{\tilde{M}}_2-\underline{\tilde{\beta}}_3\right)
\end{array}$$

Corollary 4:

The ends of the limits of the fuzzy foldings and fuuzy retractions of the system will be

$$\begin{array}{ccc}
\left(\tilde{M}^4-\tilde{\beta}_2\right) & \xrightarrow{\tilde{r}_k} & \left(\tilde{M}_1-\tilde{\beta}_1\right)\\
\lim\limits_{m\to\infty}\tilde{\Im}_{m_1}\downarrow & & \downarrow \lim\limits_{m\to\infty}\tilde{\Im}_{m_2}\\
\left(\tilde{M}_1-\tilde{\beta}_1\right) & \xrightarrow{\tilde{r}_J} & 0
\end{array}$$

$$\begin{array}{ccc}
\left(\bar{\tilde{M}}^4-\bar{\tilde{\beta}}_2\right) & \xrightarrow{\bar{\tilde{r}}_k} & \left(\bar{\tilde{M}}_1-\bar{\tilde{\beta}}_1\right)\\
\downarrow \lim\limits_{m\to\infty}\bar{\tilde{\Im}}_{m_1} & & \downarrow \lim\limits_{m\to\infty}\bar{\tilde{\Im}}_{m_2}\\
\left(\bar{\tilde{M}}_1-\bar{\tilde{\beta}}_1\right) & \xrightarrow[\bar{\tilde{r}}_J]{} & 0
\end{array}
\qquad
\begin{array}{ccc}
\left(\underline{\tilde{M}}^4-\underline{\tilde{\beta}}_2\right) & \xrightarrow{\underline{\tilde{r}}_k} & \left(\underline{\tilde{M}}_1-\underline{\tilde{\beta}}_1\right)\\
\downarrow \lim\limits_{m\to\infty}\bar{\tilde{\Im}}_{m_2} & & \downarrow \lim\limits_{m\to\infty}\underline{\tilde{\Im}}_{m_2}\\
\left(\underline{\tilde{M}}^4-\underline{\tilde{\beta}}_1\right) & \xrightarrow[\underline{\tilde{r}}_J]{} & 0
\end{array}$$

Corollary 5:

The minimum retraction of $\tilde{M}^4$ is a one or two chains of points up and down the density function η_1 and η_2. See Figure (2).

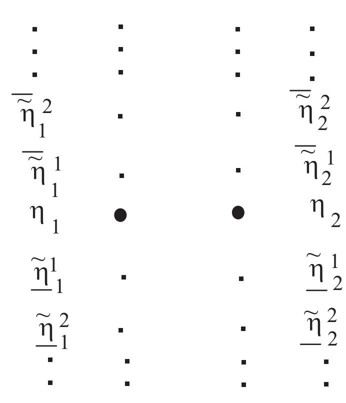

Figure 2.

Theorem 7:

If the fuzzy retraction of the hypersurface $\tilde{M}_1$ is $\tilde{\psi}: \tilde{M}_1 \to \tilde{S}_1^3$, the inclusion maps of $\tilde{M}_1$ is $\tilde{i}: \tilde{M}_1 \to \tilde{M}^4$, and of $\tilde{S}_1^3$ is $\tilde{J}: \tilde{S}_1^3 \to \tilde{M}_2$. Then there are induces fuzzy retractions such that the following diagram is commutative.

$$\begin{array}{ccccc}
 & & id_{\tilde{M}_1} & & \\
\tilde{M}_1 & \xrightarrow{\tilde{i}} & \tilde{M}^4 & \xrightarrow{\tilde{r}} & \tilde{M}_1\\
\tilde{\psi}\downarrow & & \downarrow\tilde{g} & & \downarrow\tilde{\psi}\\
\tilde{S}_1^3 & \xrightarrow{\tilde{J}} & \tilde{M}_2 & \xrightarrow{\tilde{s}} & \tilde{S}_1^3\\
 & & id_{\tilde{S}_1^3} & &
\end{array}$$

Proof:

Let the fuzzy retraction map of the hypersurface $\tilde{M}_1$ is $\tilde{\psi}: \tilde{M}_1 \to \tilde{S}_1^3$, the inclusion map of $\tilde{M}_1$ is $\tilde{i}: \tilde{M}_1 \to \tilde{M}^4, \tilde{M}_1 \subset \tilde{M}^4$, $\tilde{J}: \tilde{\psi}(\tilde{M}_1) \to \tilde{M}_2$ the fuzzy retraction of $\tilde{i}(\tilde{M}_1)$ is $\tilde{r}: \tilde{i}(\tilde{M}_1) \to \tilde{M}_1$, the fuzzy retraction of $\tilde{J}(\tilde{\psi}(\tilde{M}_1))$ is given by $\tilde{s}: \tilde{J}(\tilde{\psi}(\tilde{M}_1)) \to \tilde{S}_1^3$, and $\tilde{\psi}: \tilde{r}(\tilde{i}(\tilde{M}_1)) \to \tilde{S}_1^3$.

Theorem 8:

If the fuzzy retraction of the hypersurface $\tilde{M}_1$ is $\tilde{\psi}: \tilde{M}_1 \to \tilde{S}_1$, the inclusion maps of $\tilde{M}_1$ is $\tilde{i}: \tilde{M}_1 \to \tilde{M}^4$, and inclusion map of $\tilde{S}_1$ is $\tilde{J}: \tilde{S}_1 \to \tilde{M}_2$. Then there are induces fuzzy retractions such that the following diagram is commutative.

$$\begin{array}{ccccc}
\tilde{M}_1 & \xrightarrow{\tilde{i}} & \tilde{M}^4 & \xrightarrow{\tilde{r}} & \tilde{M}_1\\
\tilde{\psi}\downarrow & & \tilde{g}\downarrow & & \downarrow\tilde{\psi}\\
\tilde{S}_1 & \xrightarrow{\tilde{J}} & \tilde{M}_2 & \xrightarrow{\tilde{s}} & \tilde{S}_1
\end{array}$$

Proof:

Let the fuzzy retraction map of the hypersurface $\tilde{M}_1$ is $\tilde{\psi}: \tilde{M}_1 \to \tilde{S}_1$, the inclusion map of $\tilde{M}_1$ is $\tilde{i}: \tilde{M}_1 \to \tilde{M}^4$, $\tilde{M}_1 \subset \tilde{M}^4$, $\tilde{J}: \tilde{\psi}\left(\tilde{M}_1\right) \to \tilde{M}_2$ the fuzzy retraction map of $\tilde{i}\left(\tilde{M}_1\right)$ is $\tilde{r}: \tilde{i}\left(\tilde{M}_1\right) \to \tilde{M}_1$, the fuzzy retraction map of $\tilde{J}\left(\tilde{\psi}\left(\tilde{M}_1\right)\right)$ is given by $\tilde{s}: \tilde{J}\left(\tilde{\psi}\left(\tilde{M}_1\right)\right) \to \tilde{S}_1$, and $\tilde{\psi}: \tilde{r}\left(\tilde{i}\left(\tilde{M}_1\right)\right) \to \tilde{S}_1$. Hence, $\left(\tilde{s} \circ \tilde{J}\right) \circ \tilde{\psi} = \left(\tilde{\psi} \circ \tilde{r}\right) \circ \tilde{i}$.

Corollary 6:

The relations between the fuzzy retractions and the inclusion maps discussed from the following commutative diagram

$$\begin{array}{ccccc} \tilde{M}_1 & \xrightarrow{\tilde{i}} & \tilde{M}^4 & \xrightarrow{\tilde{r}} & \tilde{M}_1 \\ \tilde{\psi}\downarrow & & \tilde{g}\downarrow & & \downarrow\tilde{\phi} \\ \tilde{S}_1^3 & \xrightarrow{\tilde{j}} & \tilde{M}_1 & \xrightarrow{\tilde{s}} & \tilde{S}_1 \end{array}$$

Proof:

Let the fuzzy retraction map of the hypersurface $\tilde{M}_1$ is $\tilde{\psi}:\tilde{M}_1 \to \tilde{S}_1^3$ the inclusion maps of $\tilde{M}_1$ are $\tilde{i}(\tilde{M}_1) \to \tilde{M}^4$, $\tilde{M}_1 \subset \tilde{M}^4$, $\tilde{j}:\tilde{\psi}(\tilde{M}_1) \to \tilde{M}_1$, the fuzzy retraction of $\tilde{i}(\tilde{M}_1)$ is $\tilde{r}:\tilde{i}(\tilde{M}_1) \to \tilde{M}_1$ the fuzzy retraction map of $\tilde{j}(\tilde{\psi}(\tilde{M}_1))$ is given by $\tilde{s}:\tilde{j}(\tilde{\psi}(\tilde{M}_1)) \to \tilde{S}_1$, and $\tilde{\phi}:\tilde{r}(\tilde{i}(\tilde{M}_1)) \to \tilde{S}_1$.

3. Applications

1- The stream function of the acoustic gravity tripolar vortices is generalized to permit a study of the Earth's atmosphere under complex meteorological conditions, characterized by sheared horizontal flows and parabolic density and pressure profiles [1, 7]. See Fig. (3).

2- Consider the flow of the fluid inside a tube [3]. If we represent the velocity of the fluid as a membership degree $\mu \in [0,1]$, then $\mu=\mathbf{1}$, in the mid of the medium where the velocity of the fluid takes a maximum and is symmetric round this line but at the edge of the tube the velocity of the fluid vanishes, i.e., $\mu=\mathbf{1}$.

3- The Ritz variational method [6] during the calculation of the ground – state energy in a fuzzy framework. Consider a Hamilton H, and an arbitrary square integrable function $\mathbf{\Psi}$, so that $\langle \mathbf{\Psi}/\mathbf{\Psi} \rangle = 1$. Considering $\mathbf{\Psi}$ as a fuzzy function and the ranking system as defined in [6], similar to [6] it can be shown that $\langle \Psi/H/\Psi \rangle$ is a fuzzy upper bound on E_0 (ground-stat energy). Now $\langle \Psi/H/\Psi \rangle$ should be minimizing the distance between E_0 and respect to a number of parameters $(\alpha_1, \alpha_2, \ldots)$. This can be done by minimizing distance between E_0 and $\langle \Psi/H/\Psi \rangle$. The rest of the discussion is the same as that provided in [6].

4. Conclusion

In this paper we achieved the approval of the important of the curves and surface in fuzzy Space Time $\tilde{M}^4$ by using some geometrical transformations. The relations between fuzzy folding, fuzzy retractions and fuzzy deformation retract in fuzzy Space Time $\tilde{M}^4$ are discussed. New types of the tangent space $T_p(\tilde{M}_4)$ in fuzzy Space Time $\tilde{M}^4$ are deduced.

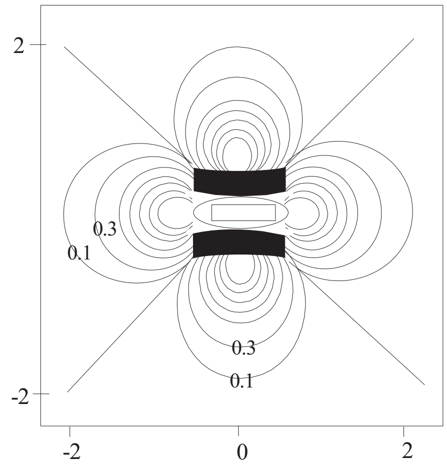

Figure 3. The stream function of the acoustic gravity tripolar [23].

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