# Statistically efficient control scheme for undersea surveillance for sonar ranging and detection 

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#### Abstract

Background/Objectives: Altered Gain Extended Kalman Filter (MGEKF) created by Song and Speyer was ended up being reasonable calculation for edges of just uninvolved target following applications in air. As of late, roughly changed increases are exhibited, which are numerically steady and precise. In this paper, this enhanced MGEKF calculation is investigated for submerged applications with a few adjustments. Methods/Statistical analysis: In submerged, the clamor in the estimations is high, turning rate of the stages is low and speed of the stages is additionally low when contrasted and the rockets in air. Findings: These attributes of the stage are considered in detail and the calculation is changed appropriately to track applications in submerged. Monte-Carlo mimicked comes about for two regular situations are exhibited with the end goal of clarification. Application/Improvements: From the outcomes it is watched that this calculation is particularly reasonable for this nonlinear edges just latent target following.


Keywords: estimation, sonar, Kalman filter, simulation, modified gain, angles-only target tracking

## 1. INTRODUCTION

In the sea environment, three dimensional edges just target movement investigations (TMA) is for the most part utilized. A spectator screens loud sonar direction and heights from an emanating target, which is thought to go in a consistent course with uniform speed. The estimations are separated from a solitary eyewitness and the onlooker forms these estimations to discover target movement parameters-Viz., extend, course, bearing, height and speed of the objective. Here the estimations are nonlinear; making the entire procedure nonlinear. Nonetheless, the changed increase amplified kalman channel (MGEKF) created by Song and Speyer [1], is the successful contributions in this field. MGEKF performs better than EKF as well as pseudo measurement filter. However the modified gain functions were derived based on the pseudo measurements. Recently the modified gain functions are improved and presented [2], in which formula of the bearing measurement is the same as in [1] and that of elevation measurement is more accurate than the original. So far, the angles in azimuth alone are considered.. Now elevation angles are also considered. A point $P$, as shown in Figure 1 whose elements are $x, y, z$.

Figure 1. Target-observer geometry


A line from $P$ is drawn on to $x y$ plane. This line is parallel to $Z$ axis. Let this line touch $x y$ plane at $P^{\prime}(x, y)$. Let the angle of elevation, $\phi$, be defined as the angle between + ve $Z$ axis (or $z u p$ ) and the line OP. Let the azimuth angle be the angle between True north and the line OP'. In Figure OPP' is denoted as $\phi$. We can write the following equations.
from $\Delta$ OPP' $^{\prime}, \sin \phi=\frac{r_{x y}}{r} \quad \cos \phi=\frac{r_{z}}{r}$
That is $\mathrm{r}_{\mathrm{z}}=\mathrm{r} \cos \phi$
$r_{x y}=r \sin \phi$
Where $r$ is the distance from point 0 to $P$ (in three dimensional space)
$r_{x y}$ is the distance from point 0 to $P^{\prime}$ ( in two dimensional space )
Also $\frac{r_{y}}{r_{x y}}=\cos B$ and $\quad \frac{r_{x}}{r_{x y}}=\sin B$

$$
\begin{equation*}
r_{y}=r_{x y} \cos B \quad r_{x}=r_{x y} \sin B \tag{2}
\end{equation*}
$$

Substituting (1) in (2)

$$
\begin{equation*}
r_{x}=r \sin \phi \sin B \quad r_{y}=r \sin \phi \cos B \tag{3}
\end{equation*}
$$

and $\quad r_{\mathrm{Z}}=\mathrm{r} \cos \phi$
where $r=\sqrt{r_{x}^{2}+r_{y}^{2}+r_{z}^{2}}=\sqrt{r_{x y}^{2}+r_{z}^{2}}$

$$
\begin{equation*}
r_{x y}=\sqrt{r_{x}^{2}+r_{y}^{2}} \tag{4}
\end{equation*}
$$

and $\frac{r_{\mathrm{x}}}{\mathrm{r}_{\mathrm{y}}}=\frac{\sin B}{\cos B}=\tan B$
$\therefore$ Bearing, $B=\tan ^{-1} \frac{r_{X}}{r_{y}}$
$\tan \phi=\frac{\mathrm{r}_{\mathrm{xy}} / \mathrm{r}}{\mathrm{r}_{\mathrm{z}} / \mathrm{r}}=\frac{\mathrm{r}_{\mathrm{xy}}}{\mathrm{r}_{\mathrm{z}}} \quad\{$ from (1) $\}$
$\phi=\tan ^{-1} \frac{\mathrm{r}_{\mathrm{xy}}}{\mathrm{r}_{\mathrm{z}}}$
Let the measurement be $\mathrm{Z}=\left[\begin{array}{l}\mathrm{B}_{\mathrm{m}} \\ \phi_{m}\end{array}\right]=\left[\begin{array}{l}\tan ^{-1} \frac{\mathrm{r}_{\mathrm{x}}}{\mathrm{r}_{\mathrm{y}}}+\sigma_{\mathrm{B}} \\ \tan ^{-1} \frac{\mathrm{r}_{\mathrm{xy}}}{\mathrm{r}_{\mathrm{z}}}+\sigma_{\phi}\end{array}\right]$
Considering the state vector as $X_{s}=\left[\begin{array}{lllllll}\& & \& & \& & & & \\ X & y & Z & r_{x} & r_{y} & r_{z}\end{array}\right]^{T}$
Then $\mathrm{H}=\frac{\partial \mathrm{h}\left(\mathrm{X}_{\mathrm{s}}\right)}{\partial \mathrm{X}_{\mathrm{s}}}=\left[\begin{array}{cccccc}\frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{K}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{k}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{Z}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{r}_{\mathrm{x}}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{r}_{\mathrm{y}}} & \frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{r}_{\mathrm{z}}} \\ \frac{\partial \mathrm{h}(\phi)}{\partial \mathrm{E}} & \frac{\partial \mathrm{h}(\phi)}{\partial \mathrm{K}} & \frac{\partial \mathrm{h}(\phi)}{\partial \mathrm{E}} & \frac{\partial \mathrm{h}(\phi)}{\partial \mathrm{r}_{\mathrm{x}}} & \frac{\partial \mathrm{h}(\phi)}{\partial \mathrm{r}_{\mathrm{y}}} & \frac{\partial \mathrm{h}(\phi)}{\partial \mathrm{r}_{\mathrm{z}}}\end{array}\right]$

$\frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{r}_{\mathrm{x}}}=\left[\frac{1}{1+\mathrm{r}_{\mathrm{x}}{ }^{2} / \mathrm{r}_{\mathrm{y}}{ }^{2}}\left\{\frac{1}{\mathrm{r}_{\mathrm{y}}}\right\}\right]=\frac{\mathrm{r}_{\mathrm{y}}^{2}}{\mathrm{r}_{\mathrm{xy}}^{2}} \cdot \frac{1}{\mathrm{r}_{\mathrm{y}}}=\frac{\mathrm{r}_{\mathrm{y}}}{\mathrm{r}_{\mathrm{xy}}^{2}}=\frac{\mathrm{r}_{\mathrm{y}}}{\mathrm{r}_{\mathrm{xy}}} \cdot \frac{1}{\mathrm{r}_{\mathrm{xy}}}=\frac{\cos \hat{B}}{\hat{\mathrm{r}}_{\mathrm{xy}}}$
$\frac{\partial h(B)}{\partial r_{y}}=\frac{r_{y}^{2}}{r_{x y}^{2}} \cdot r_{x}\left(-\frac{1}{r_{y}^{2}}\right)=\frac{-r_{x}}{r_{x y}^{2}}=\frac{-r_{x}}{r_{x y}} \frac{1}{r_{x y}}=\frac{-\sin \hat{B}}{\hat{r}_{x y}}$
$\frac{\partial \mathrm{h}(\mathrm{B})}{\partial \mathrm{r}_{\mathrm{z}}}=0$

$$
\begin{align*}
& \frac{\partial \mathrm{h}(\phi)}{\partial \mathrm{r}_{\mathrm{x}}}=\left[\frac{1}{1+\mathrm{r}_{\mathrm{xy}}^{2} / \mathrm{r}_{\mathrm{z}}^{2}}\left\{\frac{1}{\mathrm{r}_{\mathrm{z}}} \frac{\partial \mathrm{r}_{\mathrm{xy}}}{\partial \mathrm{x}}\right\}\right]=\frac{\mathrm{r}_{\mathrm{z}}^{2}}{\mathrm{r}^{2}} \cdot \frac{1}{\mathrm{r}_{\mathrm{z}}} \frac{\partial \sqrt{\mathrm{r}_{\mathrm{x}}^{2}+\mathrm{r}_{\mathrm{y}}^{2}}}{\partial \mathrm{r}_{\mathrm{z}}}=\frac{\mathrm{r}_{\mathrm{z}}}{\mathrm{r}^{2}} \cdot \frac{2 \mathrm{r}_{\mathrm{x}}}{2 \mathrm{r}_{\mathrm{xy}}} \\
& \\
& =\frac{\mathrm{r}_{\mathrm{z}}}{\mathrm{r}^{2}} \frac{\mathrm{r}_{\mathrm{x}}}{\mathrm{r}_{\mathrm{xy}}}=\frac{\cos \hat{\phi} \sin \hat{\mathrm{B}}}{\hat{\mathrm{r}}} \\
& \frac{\partial \mathrm{~h}(\phi)}{\partial \mathrm{r}_{\mathrm{y}}}=\frac{\mathrm{r}_{\mathrm{z}}}{\mathrm{r}^{2}} \cdot \frac{1}{\mathrm{r}_{\mathrm{z}}} \cdot \frac{2 \mathrm{r}_{\mathrm{y}}}{2 \mathrm{r}_{\mathrm{xy}}}=\frac{\mathrm{r}_{\mathrm{z}}}{\mathrm{r}^{2}} \cdot \frac{\mathrm{r}_{\mathrm{y}}}{\mathrm{r}_{\mathrm{xy}}}=\frac{\cos \hat{\phi} \cos \hat{\mathrm{B}}}{\hat{\mathrm{r}}} \\
& \frac{\partial \mathrm{~h}(\phi)}{\partial \mathrm{r}_{\mathrm{z}}}=\frac{\mathrm{r}_{\mathrm{z}}^{2}}{\mathrm{r}^{2} \mathrm{r}_{\mathrm{xy}}\left(-\frac{1}{\mathrm{r}_{\mathrm{z}}^{2}}\right) \cdot=\frac{-\mathrm{r}_{\mathrm{xy}}}{\mathrm{r}^{2}}=\frac{-\mathrm{r} \sin \phi}{\mathrm{r}^{2}}=\frac{-\sin \hat{\phi}}{\hat{\mathrm{r}}}}  \tag{10}\\
& \left.\therefore \mathrm{H}=\left[\begin{array}{lllll}
0 & 0 & 0 & \frac{\cos \hat{\mathrm{~B}}}{\hat{r}_{\mathrm{xy}}} & -\frac{\sin \hat{\mathrm{B}}}{\hat{r}_{\mathrm{xy}}} \\
0 & 0 & 0 & \frac{\cos \hat{\phi} \sin \hat{\mathrm{~B}}}{\hat{\mathrm{r}}} & \frac{\cos \hat{\phi} \cos \hat{\mathrm{~B}}}{\hat{\mathrm{r}}}
\end{array}\right]-\frac{\sin \hat{\phi}}{\hat{\mathrm{r}}}\right]
\end{align*}
$$

Horizontal plane and bearing measurements
If the range in horizontal plane is $\sqrt{\mathrm{r}_{\mathrm{x}}{ }^{2}+\mathrm{r}_{\mathrm{y}}{ }^{2}}$, then the estimated range be

$$
\begin{equation*}
\hat{r}_{x y}=\sqrt{r_{x}^{2}+r_{y}^{2}} \tag{11}
\end{equation*}
$$

$$
\begin{array}{lrl}
\text { As } & r_{x}=r_{x y} \sin B & \hat{r}_{x}=\hat{r}_{x y} \sin \hat{B} \\
r_{y}=r_{x y} \cos B & \hat{r}_{y}=\hat{r}_{x y} \cos \hat{B} \\
r_{x} \sin B+r_{y} \cos B=r_{x y} \sin ^{2} B+r_{x y} \cos ^{2} B= & r_{x y} & \\
\hat{r}_{x} \sin B+\hat{r}_{y} \cos B=\hat{r}_{x y} & \tag{13}
\end{array}
$$

By adding $\mathrm{r}_{\mathrm{xy}}+\hat{\mathrm{r}}_{\mathrm{xy}}$
$r_{x y}+\hat{r}_{x y}=r_{x} \sin B+r_{y} \cos B+\hat{r}_{x} \sin \hat{B}+\hat{r}_{y} \sin \hat{B}$
adding both sides $-r_{x} \sin \hat{B}-\hat{r}_{x} \sin \hat{B}-\hat{r}_{y} \cos B+r_{y} \cos \hat{B}$ to the above equation
$r_{x y}+\hat{r}_{x y}-r_{x} \sin \hat{B}-\hat{r}_{x} \sin B-r_{y} \cos B-r_{y} \cos \hat{B}=\sin B\left(r_{x}-\hat{r}_{x}\right)+\cos B\left(r_{y}-\hat{r}_{y}\right)-\sin \hat{B}\left(r_{x}-\hat{r}_{x}\right)-\cos \hat{B}\left(r_{y}-\hat{r}_{y}\right)$

$$
\begin{equation*}
=\left(r_{x}-\hat{r}_{x}\right)(\sin B-\sin \hat{B})+\left(r_{y}-\hat{r}_{y}\right)(\cos B-\cos \hat{B}) \tag{14}
\end{equation*}
$$

Substituting for $r_{x}, \hat{r}_{x}$ and $r_{y}, \hat{r}_{y}$ on L.H.S of (14)
$\mathrm{r}_{\mathrm{xy}}+\hat{\mathrm{r}}_{\mathrm{xy}}-\mathrm{r}_{\mathrm{xy}} \sin B \sin \hat{B}-\hat{r}_{\mathrm{xy}} \sin \hat{B} \sin B-\hat{r}_{\mathrm{xy}} \cos \hat{B} \cos B-\hat{\mathrm{r}}_{\mathrm{xy}} \cos B \cos \hat{B}=$ R.H.S of (14)
$\left(r_{x y}+\hat{r}_{x y}\right)-\sin B \sin \hat{B}\left(r_{x y}-\hat{r}_{x y}\right)-\cos B \cos \hat{B}\left(r_{x y}-\hat{r}_{x y}\right)=$ R.H.S of (14)
$\left(\mathrm{r}_{\mathrm{xy}}+\hat{\mathrm{r}}_{\mathrm{xy}}\right)(1-\sin B \sin \hat{\mathrm{~B}}-\cos \mathrm{B} \cos \hat{\mathrm{B}})=$ R.H.S of (14)
$\left(r_{x y}+\hat{r}_{x y}\right)(1-\cos (B-\hat{B}))=$ R.H.S of (14)
$r_{x y}+\hat{r}_{x y}=\frac{\left(r_{x}-\hat{r}_{x}\right)(\sin B-\sin \hat{B})+\left(r_{y}-\hat{r}_{y}\right)(\cos B-\cos \hat{B})}{1-\cos (B-\hat{B})}$
By subtracting $\mathrm{r}_{\mathrm{xy}}$ from $\hat{\mathrm{r}}_{\mathrm{xy}}$
$r_{x y}-\hat{r}_{x y}=-\hat{r}_{x} \sin \hat{B}-\hat{r}_{y} \sin \hat{B}+r_{x} \sin B+r_{y} \cos B$
Adding both sides $-\hat{r}_{x} \sin \hat{B}-\hat{r}_{x} \sin B-\hat{r}_{y} \cos B-r_{y} \cos \hat{B}$ to (16) and continuing the previous procedure
$r_{x y}-\hat{r}_{x y}=\frac{\left(r_{x}-\hat{r}_{x}\right)(\sin B+\sin \hat{B})+\left(r_{y}-\hat{r}_{y}\right)(\cos B+\cos \hat{B})}{1+\cos (B-\hat{B})}$
using (15) and (17)
$2 \hat{r}_{x y}=\left(r_{x}-\hat{r}_{x}\right)\left[\frac{\sin B-\sin \hat{B}}{1-\cos (B-\hat{B})}-\frac{\sin B+\sin \hat{B}}{1+\cos (B-\hat{B})}\right]+\left(r_{y}-\hat{r}_{y}\right)\left[\frac{\cos B-\cos \hat{B}}{1-\cos (B-\hat{B})}+\frac{\cos B+\cos \hat{B}}{1+\cos (B-\hat{B})}\right]$
(18) can be simplified as

$$
\begin{align*}
\frac{\sin B-\sin \hat{B}}{1-\cos (B-\hat{B})}-\frac{\sin B+\sin \hat{B}}{1+\cos (B-\hat{B})} & =\frac{(1+\cos (B-\hat{B})(\sin B-\sin \hat{B})-(1-\cos (B-\hat{B})(\sin B+\sin \hat{B})}{1-\cos ^{2}(B-\hat{B})}  \tag{18}\\
& =\frac{2 \sin B \cos (B-\hat{B})-2 \sin \hat{B}}{\sin ^{2}(B-\hat{B})} \\
& =\frac{2 \cos B \sin (B-\hat{B})}{\sin ^{2}(B-\hat{B})}=\frac{2 \cos B}{\sin (B-\hat{B})} \tag{19}
\end{align*}
$$

III $^{\text {ly }}\left(\mathrm{r}_{\mathrm{y}}-\hat{r}_{\mathrm{y}}\right)$ coefficient is simplified to $\frac{2 \cos (\mathrm{~B}-\hat{\mathrm{B}}) \cos \mathrm{B}-2 \cos \hat{\mathrm{~B}}}{\sin ^{2}(\mathrm{~B}-\hat{\mathrm{B}})}$

$$
\begin{equation*}
=-\frac{2 \sin B}{\sin (B-\hat{B})} \tag{20}
\end{equation*}
$$

$$
\begin{equation*}
\therefore 2 \hat{r}_{x y}=\frac{2 \cos B\left(r_{x}-\hat{r}_{x}\right)}{\sin (B-\hat{B})}-\frac{2 \sin B\left(r_{y}-\hat{r}_{y}\right)}{\sin (B-\hat{B})} \tag{21}
\end{equation*}
$$

(21) is rewritten as

$$
\begin{equation*}
\sin (B-\hat{B})=\frac{\cos B\left(r_{x}-\hat{r}_{x}\right)-\sin B\left(r_{y}-\hat{r}_{y}\right)}{\hat{r}_{x y}} \tag{22}
\end{equation*}
$$

Angle measurement

$$
\begin{array}{ll}
\tan ^{-1} \frac{r_{x}}{r_{y}}=B & \text { generates } \sin (B-\hat{B})=\frac{\cos B\left(r_{x}-\hat{r}_{x}\right)-\sin B\left(r_{y}-\hat{r}_{y}\right)}{\hat{r}_{x y}} \\
\tan ^{-1} \frac{r_{x y}}{r_{z}}=\phi & \text { generates } \sin (\phi-\hat{\phi})=\frac{\cos \phi\left(r_{x y}-\hat{r}_{x y}\right)-\sin \phi\left(r_{z}-\hat{r}_{z}\right)}{r} \tag{23}
\end{array}
$$

Where $r_{x} \rightarrow r_{x y}, \quad r_{y} \rightarrow r_{z}, B \rightarrow \phi \quad r_{x y} \rightarrow r_{x y z}(=r)$
$\mathrm{r}_{\mathrm{xy}}-\hat{\mathrm{r}}_{\mathrm{xy}}$ is given by (17) as follows
$r_{x y}-\hat{r}_{x y}=\frac{\left(r_{x}-\hat{r}_{x}\right)(\sin B+\sin \hat{B})+\left(r_{y}-\hat{r}_{y}\right)(\cos B+\cos \hat{B})}{1+\cos (B-\hat{B})}$
It is known that $\cos (p+q)+\cos (p-q)=2 \cos p \cos q$
here $p+q=B \quad p-q=\hat{B} \quad$ giving $p=\frac{B+\hat{B}}{2}$ and $\quad q=\frac{B-\hat{B}}{2}$
So $\cos \mathrm{B}+\cos \hat{\mathrm{B}}=2 \cos \frac{B+\hat{B}}{2} \cos \frac{B-\hat{B}}{2}$
III ${ }^{\text {ly }} \sin (p+q)+\sin (p-q)=2 \sin p \sin q$
$\sin B+\sin \hat{B}=2 \sin \frac{B+\hat{B}}{2} \sin \frac{B-\hat{B}}{2}$
III $^{\text {ly }} 1+\cos 2 \alpha=\frac{2 \cos ^{2} \alpha}{2}=\cos ^{2} \alpha$
so $1+\cos (B-\hat{B})=2 \cos ^{2} \frac{B-\hat{B}}{2}$
using (24), (25) and (26) , eqn. (17) becomes

$$
\begin{align*}
r_{x y}-\hat{r}_{x y}= & \frac{\left(r_{x}-\hat{r}_{x}\right) 2 \sin \frac{(B+\hat{B})}{2} \cos \frac{(B-\hat{B})}{2}+\left(r_{y}-\hat{r}_{y}\right) 2 \cos \frac{(B+\hat{B})}{2} \cos \frac{(B-\hat{B})}{2}}{2 \cos ^{2} \frac{(B-\hat{B})}{2}} \\
= & \frac{\left(r_{x}-\hat{r}_{x}\right) \sin \frac{(B+\hat{B})}{2}+\left(r_{y}-\hat{r}_{y}\right) \cos \frac{(B+\hat{B})}{2}}{\cos \frac{(B-\hat{B})}{2}} \tag{27}
\end{align*}
$$

Substituting (27) in (23)
$\sin (\phi-\hat{\phi})=\frac{\cos \phi}{\hat{r}}\left[\frac{\left(\mathrm{r}_{\mathrm{x}}-\hat{\mathrm{r}}_{\mathrm{x}}\right) \sin \frac{(\mathrm{B}+\hat{\mathrm{B}})}{2}+\left(\mathrm{r}_{\mathrm{y}}-\hat{\mathrm{r}}_{\mathrm{y}}\right) \cos \frac{(\mathrm{B}+\hat{\mathrm{B}})}{2}}{\cos \frac{(\mathrm{~B}-\hat{\mathrm{B}})}{2}}\right]-\frac{\sin \phi}{\hat{\mathrm{r}}}\left(\mathrm{r}_{\mathrm{z}}-\hat{\mathrm{r}}_{\mathrm{z}}\right)$
As $\phi-\hat{\phi}$ tend to zero and $\sin (\phi-\hat{\phi}) \rightarrow(\phi-\hat{\phi})$ and $\sin (\mathrm{B}-\hat{\mathrm{B}}) \rightarrow \quad(\mathrm{B}-\hat{\mathrm{B}}) \cdot(22)$ and (28) can be written in matrix form as

$$
\left[\begin{array}{c}
(\mathrm{B}-\hat{\mathrm{B}})  \tag{29}\\
(\phi-\hat{\phi})
\end{array}\right]=\left[\begin{array}{ccc}
\frac{\cos \mathrm{B}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & -\frac{\sin \mathrm{B}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & 0 \\
\frac{\sin \left(\frac{\mathrm{~B}+\hat{\mathrm{B}}}{2}\right) \cos \phi}{\cos \left(\frac{\mathrm{B}+\hat{\mathrm{B}}}{2}\right) \hat{\mathrm{r}}} & \frac{\cos \left(\frac{\mathrm{~B}+\hat{\mathrm{B}}}{2}\right) \cos \phi}{\cos \left(\frac{\mathrm{B}-\hat{\mathrm{B}}}{2}\right) \hat{\mathrm{r}}} & \frac{-\sin \phi}{\hat{\mathrm{r}}}
\end{array}\right]\left[\begin{array}{l}
\mathrm{r}_{\mathrm{x}}-\hat{\mathrm{r}}_{\mathrm{x}} \\
\mathrm{r}_{\mathrm{y}}-\hat{\mathrm{r}}_{\mathrm{y}} \\
\mathrm{r}_{\mathrm{z}}-\hat{\mathrm{r}}_{\mathrm{z}}
\end{array}\right]
$$

True bearing is not available, if it is replaced by measured bearing

$$
\begin{aligned}
{\left[\begin{array}{c}
(B-\hat{B}) \\
(\phi-\hat{\phi})
\end{array}\right] } & =\left[\begin{array}{ccc}
\frac{\cos B_{m}}{\hat{r}_{x y}} & -\frac{\sin B_{m}}{\hat{r}_{x y}} & 0 \\
\frac{\sin \left(\frac{B_{m}+\hat{B}}{2}\right) \cos \phi}{\frac{\cos \left(\frac{B_{m}+\hat{B}}{2}\right) \cos \phi}{\cos \left(\frac{B_{m}+\hat{B}}{2}\right) \hat{r}}} \begin{array}{c}
\cos \left(\frac{B_{m}-\hat{B}}{2}\right) \hat{r}
\end{array} & \frac{-\sin \phi}{\hat{r}}
\end{array}\right]\left[\begin{array}{l}
r_{x}-\hat{r}_{x} \\
r_{y}-\hat{r}_{y} \\
r_{z}-\hat{r}_{z}
\end{array}\right] \\
& =g\left[\begin{array}{c}
r_{x}-\hat{r}_{x} \\
r_{y}-\hat{r}_{y} \\
r_{z}-\hat{r}_{z}
\end{array}\right]
\end{aligned}
$$

Where g is given by

$$
\mathrm{g}=\left[\begin{array}{ccc}
\frac{\cos \mathrm{B}_{\mathrm{m}}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & -\frac{\sin \mathrm{B}_{\mathrm{m}}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & 0  \tag{31}\\
\frac{\sin \left(\frac{\mathrm{~B}_{\mathrm{m}}+\hat{\mathrm{B}}}{2}\right) \cos \phi}{\cos \left(\frac{\mathrm{B}_{\mathrm{m}}+\hat{\mathrm{B}}}{2}\right) \hat{\mathrm{r}}} & \frac{\cos \left(\frac{\mathrm{~B}_{\mathrm{m}}+\hat{\mathrm{B}}}{2}\right) \cos \phi}{\cos \left(\frac{\mathrm{B}_{\mathrm{m}}-\hat{\mathrm{B}}}{2}\right) \hat{\mathrm{r}}} & \frac{-\sin \phi}{\hat{\mathrm{r}}} \\
\end{array}\right]
$$

Considering $\stackrel{\&}{\mathrm{X}}, \mathrm{y}$ and $\underset{\mathrm{z}}{\&}$ also, g is given by
$\mathrm{g}=\left[\begin{array}{cccccc}0 & 0 & 0 & \frac{\cos B_{m}}{\hat{\mathrm{r}}_{\mathrm{xy}}=\hat{\mathrm{r}}_{\sin \hat{\phi}}} & \frac{-\sin \mathrm{B}_{\mathrm{m}}}{\hat{\mathrm{r}}_{\mathrm{xy}}=\hat{\mathrm{r}}_{\sin \hat{\phi}}} & 0 \\ 0 & 0 & 0 & \frac{\cos \phi \sin \left(\frac{\mathrm{~B}_{\mathrm{m}}+\hat{B}}{2}\right)}{\hat{r} \cos \left(\frac{\mathrm{~B}_{\mathrm{m}}-\hat{\mathrm{B}}}{2}\right)} & \frac{\cos \phi \cos \left(\frac{\mathrm{B}_{\mathrm{m}}+\hat{\mathrm{B}}}{2}\right)}{\hat{\mathrm{r} \cos \left(\frac{\mathrm{B}_{\mathrm{m}}-\hat{\mathrm{B}}}{2}\right)}} & -\frac{\sin \phi}{\hat{r}}\end{array}\right]$

$$
\mathrm{H}=\left[\begin{array}{cccccc}
0 & 0 & 0 & \frac{\cos \hat{B}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & -\frac{\sin \hat{\mathrm{B}}}{\hat{\mathrm{r}}_{\mathrm{xy}}} & 0  \tag{33}\\
0 & 0 & 0 & \frac{\cos \hat{\phi} \sin \hat{B}}{\hat{\mathrm{r}}} & \frac{\cos \hat{\phi} \cos \hat{B}}{\hat{\mathrm{r}}} & -\frac{\sin \hat{\phi}}{\hat{\mathrm{r}}}
\end{array}\right]
$$

Implementation of Kalman filter
It is assumed that the target is not changing depth.
Let $X_{s}=\left[\begin{array}{lllll}\& & \& & & & r^{T} \\ X & y & r_{x} & r_{y} & r_{z}\end{array}\right]^{T}$

$$
\begin{equation*}
\& \& \tag{34}
\end{equation*}
$$

Where X y are target (absolute) velocity components
Let $\mathrm{X}(0 \mid 0)$ be $\mathrm{X}_{\mathrm{f}}(0 \mid 0)$
$\mathrm{X}_{\mathrm{f}}[0 \mid 0]=\left[\begin{array}{llll}10 & 10 & 10 & 15000 \\ \sin \mathrm{~B}_{\mathrm{m}} \sin \phi_{\mathrm{m}} & 15000 \cos _{\mathrm{m}} \sin \phi_{\mathrm{m}} & 15000 \cos \phi_{\mathrm{m}}\end{array}\right]^{\mathrm{T}}(35)$
$\mathrm{P}(0 \mid 0)=1$
$\mathrm{G}(\mathrm{k}+1)=\mathrm{P}(\mathrm{k}+1 \mid \mathrm{k}) \mathrm{H}^{\mathrm{T}}(\mathrm{k}+1)\left[\mathrm{H}(\mathrm{k}+1) \mathrm{P}(\mathrm{k}+1 \mid \mathrm{k}) \mathrm{H}^{\mathrm{T}}(\mathrm{k}+1)+\mathrm{r}(\mathrm{k}+1)\right]$
Where $r(k+1)=\left[\begin{array}{cc}\sigma{ }_{\mathrm{B}}^{2}(\mathrm{k}+1) & 0 \\ 0 & \sigma_{\phi}^{2}(\mathrm{k}+1)\end{array}\right]$
Where $\sigma_{\mathrm{B}}^{2}$ and $\sigma_{\phi}^{2}$ are input error bearing and elevation measurement covariances respectively
$\hat{\mathrm{X}}(\mathrm{k}+1 \mid \mathrm{k}+1)=\hat{\mathrm{X}}(\mathrm{k}+1 \mid \mathrm{k})+\mathrm{k}(\mathrm{k}+1)[\mathrm{Z}(\mathrm{k}+1)-\mathrm{h}\{\hat{\mathrm{x}}(\mathrm{k}+1 \mid \mathrm{k})\}]$
where $Z(k+1)=\left[\begin{array}{l}B_{m} \\ \phi_{\mathrm{m}}\end{array}\right]$
$P(k+1 \mid k+1)=(I-G g) P(I-G g) P(I-G g)^{T}+G r G^{T}$

$$
\begin{equation*}
=(\mathrm{I}-\mathrm{G}(\mathrm{k}+1) \mathrm{g}) \mathrm{P}(\mathrm{k}+1 \mid \mathrm{k})(\mathrm{I}-\mathrm{G}(\mathrm{k}+1) \mathrm{g})^{\mathrm{T}}+\mathrm{G}(\mathrm{k}+1) \mathrm{r}(\mathrm{k}+1) \mathrm{G}^{\mathrm{T}}(\mathrm{k}+1)( \tag{41}
\end{equation*}
$$

For next cycle $\hat{\mathrm{x}}(\mathrm{k} \mid \mathrm{k})=\hat{\mathrm{x}}(\mathrm{k}+1 \mid \mathrm{k}+1)$
and $P(k \mid k)=P(k+1 \mid k+1)$
$\hat{x}(\mathrm{k}+1 \mid \mathrm{k})=\phi(\mathrm{k}+1 \mid \mathrm{k}) \hat{\mathrm{x}}(\mathrm{k} \mid \mathrm{k})+\mathrm{B}$
where $B=\left[\begin{array}{c}0 \\ 0 \\ 0 \\ -\left\{\mathrm{x}_{0}(\mathrm{k}+1)-\mathrm{x}_{0}(\mathrm{k})\right\} \\ -\left\{\mathrm{y}_{0}(\mathrm{k}+1)-\mathrm{y}_{0}(\mathrm{k})\right\} \\ -\left\{\mathrm{z}_{0}(\mathrm{k}+1)-\mathrm{z}_{0}(\mathrm{k})\right\}\end{array}\right]$

$$
\begin{equation*}
\mathrm{P}(\mathrm{k}+1 \mid \mathrm{k})=\phi(\mathrm{k}+1 \mid \mathrm{k}) \mathrm{P}(\mathrm{k} \mid \mathrm{k}) \phi^{\mathrm{T}}(\mathrm{k}+1 \mid \mathrm{k})+\mathrm{Q}(\mathrm{k}+1) \tag{46}
\end{equation*}
$$

Where Q is plant covariance matrix
A maneuvering target and tracking using bearing and elevation measurements
$\underset{X}{\&}(k+1)=\stackrel{\&}{X}(k)+t_{X}^{\&}(k)$
$\int_{X}^{\&}(k+1)=\int_{0}^{t} \underset{X}{\&}(k) d z+\int_{0}^{t} \&{ }_{x}^{\&}(k) d z$
$x(k+1)-x(k)=\stackrel{\&}{x}(k) \cdot t+\frac{t^{2}}{2} \&_{x}^{\&}(k)$
$\mathrm{x}(\mathrm{k}+1)=\mathrm{x}(\mathrm{k})+\mathrm{t} \stackrel{\&}{\mathrm{x}}(\mathrm{k})+\frac{\mathrm{t}^{2}}{2} \&_{\mathrm{x}} \&_{(\mathrm{k})}$
This can be easily remembered as

$$
\begin{aligned}
& X\left(\mathrm{t}_{\mathrm{n}}+\varsigma\right)=X\left(\mathrm{t}_{\mathrm{n}}\right)+\varsigma \mathrm{X}\left(\mathrm{t}_{\mathrm{n}}\right)+\frac{\varsigma^{2}}{2} \&_{\mathrm{X}} \&_{\left(\mathrm{t}_{\mathrm{n}}\right)} \\
& \mathrm{x}(\mathrm{Kt}+\mathrm{t})=\mathrm{x}(\mathrm{Kt})+\mathrm{t} \mathrm{X}(\mathrm{kt})+\frac{\mathrm{t}^{2}}{2} \&_{\mathrm{X}} \&_{(\mathrm{kt})}
\end{aligned}
$$

or simply
$\mathrm{x}(\mathrm{k}+1)=\mathrm{x}(\mathrm{k})+\mathrm{t} \underset{\mathrm{X}}{\mathrm{X}}(\mathrm{k})+\frac{\mathrm{t}^{2}}{2} \&_{\mathrm{x}} \&_{(\mathrm{k})}$

## 1. Implementation of the algortihm for underwater application

The above mentioned improved MGEKF algorithm is implemented for underwater passive target tracking as follows. In underwater, the variance of the noise in the measurements is very high and so the measurements are preprocessed (averaging the measurements over some duration, say 20 seconds) to reduce the variance of the noise in the measurements[3-15]. Hence, though the measurements are available every one second, the update of the solution is presented at every 20 seconds. This does not hamper the results as the vehicles move in water at very low speeds when compared with that of in air. The initial target state vector is chosen as follows. As only bearing and elevation measurements are available and there is no way to guess the velocity components of the target, these components are each assumed to be $10 \mathrm{~m} / \mathrm{sec}$ which is very close to the realistic speed of the vehicles in underwater. The range of the day, say 15000 meters, is utilized in the calculation of initial position estimate of the target is as
$X(0 \mid 0)=[x y z x y z]^{\top}=\left[\begin{array}{lll}10101015000 * \sin B_{m}(0) * \sin m(0) \quad 15000 * \sin m(0) * \cos B_{m}(0) \quad 15000 * \cos m(0)\end{array}\right]^{\top}$ where $B_{m}(0)$ and $m(0)$ are initial bearing and elevation measurements. Initial covariance matrix is chosen according the standard procedure [16].

## 2. Simulation results

Computer algorithm is developed and tested with simulated data to illustrate the performance of this estimator. All raw bearings and elevation measurements are corrupted by additive zero mean Gaussian noise with a r.m.s level of 1 degree $\& 0.3$ respectively and then preprocessed over a period of twenty seconds. Corresponding to a tactical scenario in which the target is at the initial range of 19000 yards ( 17373.6 meters) at initial bearing and elevation of zero and 45 degrees respectively, the errors in estimates are plotted in Figure 2. (On seawaters, usually range is expressed in yards and speed in knots.) The target is assumed to be moving at a constant course of 140 degrees at a speed of 25 knots ( 12.875 meters / sec). Observer is assumed to moving at a constant speed of 7 knots ( 3.605 meters $/ \mathrm{sec}$ ) with a pitch angle of 45 degrees. Finally the results have been ensemble averaged over several Monte Carlo runs. In general the error allowed in the estimated target motion parameters in underwater is eight percent in range estimate, three degrees in course estimate and three meters/sec in velocity estimate. It is observed that this required accuracy is obtained from 240 seconds onwards and so this algorithm seems to be very much useful for underwater passive target tracking.

Figure 2. Errors in target motion parameters for given scenario



$$
1(\mathrm{a})
$$

2(b)


## 3. Conclusion

The studied scenarios show that the algorithm is good for target tracking from a hovering object through which sonar is sent to track the target. Hence it is also used in underwater applications. Modified version of Extended Kalman filter is used in this algorithm to estimate range and bearing measurements of underwater targets. It is observed from taken scenarios that the errors are small and can be settled down easily.

## 4. References

1. P.J. Golkoki, M.A. Islam. An alternative derivation of the modified gain function of song and speyer. IEEE Transactions on Automatic Control. 1999, 36(11), 1223-1326.
2. M. Longbin, L. Qi, Z. Yiyu, S. Zhongkang. Utilization of the universal linearization in target tracking. IEEE Transactions on Aerospace and Electronic Systems.1997, 2, 941-945.
3. T.L. Song, J.L. Speyer. A stochastic analysis of a modified gain extended kalman filter with applications to estimation with bearing only measurements. IEEE Transactions on Automatic Control.1958, 30(10), 940-949.
4. Jawahar, S.K. Rao. Modified polar extended kalman filter (MP-EKF) for bearings only target tracking. Indian Journal of Science and Technology. 2016, 9(26), 0974-5645.
5. Y.T. Chan, S.W. Rudnicki. Bearings only doppler bearing tracking using instrumental variables. IEEE Transactions on aerospace and electronic Systems. 1992, I 28(4), 1076-1083.
6. A. Jawahar, S. Koteswara Rao. Modified Polar Extended Kalman Filter (MP-EKF) for bearings only target tracking. Indian Journal of Science and Technology. 2016, 9 (26), 1-5.
7. Jawahar, S. Koteswara Rao, A.S.D. Murthy, K.S. Srikanth, R.P. Das. Advanced submarine integrated weapon control system. Indian Journal of Science and Technology. 2015, 8(35), 1-3.
8. Jawahar, S. Koteswara Rao, A.S.D. Murthy, K.S. Srikanth, R.P. Das. Underwater passive target tracking in constrained environment. Indian Journal of Science and Technology. 2015, 8(35), 1-4.
9. Jawahar, S. Koteswara Rao, S.K.B. Karishma. Target estimation analysis using data association and fusion. International Journal of Oceans and Oceanography. 2015, 9(2), 203-210.
10. Jawahar, S. Koteswara Rao. Recursive multistage estimator for bearings only passive target tracking in ESM EW Systems. Indian Journal of Science and Technology. 2015, 8(26), 1-5.
11. Jawahar. Feasible course trajectories for undersea sonar target tracking systems. Indian Journal of Automation and Artificial Intelligence. 2016, 3(1), 1-5.
12. Jawahar, V.C. Chakravarthi. Improved Nonlinear Signal Estimation Technique For Undersea Sonar-Based Naval Applications. Innovare Journal of Engineering and Technology. 2016, 4(4), 20-25.
13. Jawahar A, V.C. Chakravarthi. Comparative analysis of non linear estimation schemes used for undersea sonar applications. Innovare Journal of Engineering and Technology. 2016, 4 (4), 14-19.
14. K.L. Prasanna, S. Koteswara Rao, B.O.L. Jagan, A. Jawahar, S.K.B. Karishma Data Fusion in Underwater Environment. International Journal of Engineering and Technology (IJET). 2016, 8(1), 225-234.
15. K.L. Prasanna, S. Koteswara Rao, A. Jawahar, S.K.B. Karishma. Modern Estimation technique for undersea active target tracking. International Journal of Engineering and Technology. 2016, 8(2), 791-803.
16. K.L. Prasanna, S. Koteswara Rao, A. Jawahar, S.K.B. Karishma. Ownship Strategies during Hostile Torpedo Attack. Indian Journal of Science and Technology. 2016, 9 (16), 1-5.

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