Visual Simulation of Explosion Effects Based on Mathematical Model and Particle System

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Abstract—The paper firstly analyzed the factors related to the smoke diffusion and dissipation when explosion, and the mathematical model of smoke diffusion and dissipation are studied. Then the approach of particle system is used to management the movement of smoke particles. At last, the texture mapping techniques and billboard techniques are used to rendering the smoke. The results show that this method for visual simulation of exploding in virtual environment is efficiently and realistic.

Index Terms—Mathematical Models; Smoke Simulation; Particle Systems; Texture Mapping

I. INTRODUCTION

With the development of computer graphics technology, the scenes of smoke, flame, explosions, cloud, snow and other dynamic irregular fuzzy objects appear more and more in virtual battlefield environment. In virtual battlefield environment, the explosion effect is an important element, whether battlefield environment is realistic largely affects the trainer’s training effect. But due to the dynamic irregular geometry and uncertainty internal of smoke, which makes the classical Euclidean geometry becoming powerless. Meanwhile, the movement of smoke in the combustion process is subjected to various internal and external factors, so the general modeling approach is difficult to describe it.

In recent decades, computer graphics researchers have made a lot of smoke simulation methods, some of which is effect, and the application fields also are larger. But for various reasons, the method which considered satisfactory generally is still relatively limited; it is difficult to say which method is more suitable for people’s needs. Thus for computer workers, especially computer graphics researchers, the development of new and more convenient way to simulate the irregular fuzzy objects like smoke and so on is essential, and combining the existing methods organically to meet the needs of the times is still an important issue.

A. The Previous Work

Commonly, there are three methods for such dynamic irregular fuzzy object’s modeling. There are mathematical and physics-based model, Particle system model and texture mapping method [1] [2] [3]. Particle system is relatively simple, it is easy to describe the performance of smoke’s spread and disappeared details. Especially, the random changes in performance are relatively easy, but because of the movement process is too random, we can not describe the changes of the smoke illumination accurately. The mathematical and physics-based methods use the fluid motion equations to describe the movement of the smoke. Its advantage is scientific and reasonable, so the effect of the visual simulation is real. But solving the hydrodynamic equations is an explicit method, only time step size is small enough to ensure operation stability, which also led to the operation speed is very slow, it will affect real-time. The method based on texture mapping technology has obvious advantages on speed though the application scope is limited. A simple texture image can replace a large number of particles, so it can resave a lot of computer resources and the speed is accelerated. The core of texture mapping technology is how to structure the texture images. In addition, how to make it more in accordance with people’s visual effects closely is needed to further research. The comparisons of three kind methods for smoke simulation are shown in table 1.

B. Our Work

Each method had their advantages and shortcomings, application scopes and result characteristics. Aiming to three methods above, this paper first studies the diffusion and dissipation mathematical model of smoke when explosion, then using the model of particle system to control the movement of smoke particle. The combining of particle systems and mathematical models can overcome the shortcomings of particle motion’s random. When rendering particles, we use texture mapping technology and billboard technology to improve rendering speed. The result shows that this algorithm is simple and real-time.

II. MODELING OF SHELLS EXPLODING

Smoke diffusion of explosion can be seen as a rapid diffuse process from the point source to infinite space instantaneously, Second-order parabolic partial differential equations can be used to describe the smoke concentration variation. Smoke diffusion process and the smoke concentration variation which we observed is relate to light absorption, and also relates to the sensitivity of observation instruments or the naked eye. For example, with the naked eye that the smoke is gone,
and with the instrument of high sensitivity is still observed [4] [5].

Therefore, the entire modeling process should contain: the variation of the smoke concentration; light intensity variation through the smoke; the description to identify bright and dark of instrument sensitivity; changing process of opaque zone boundary. In order to simplify the modeling, First of all, we make the following assumptions:

(A) Under ideal conditions (without considering the impact of wind and earth), the explosion of shells as a point in the air releasing instantaneous smoke, smoke spread in the infinite space.

(B) The diffusion of smoke obeys the diffusion law, i.e. the area of the flow rate is proportional to its concentration gradient.

(C) The light strength passing through the smoke is reduced according to the absorption of the smoke, the reduction of light intensity on unit distance is proportional to the concentration of smoke; the light absorption in atmosphere effect is negligible.

(D) In the diffusion process of smoke, light intensity $I_0$ which does not pass through the smoke directly into the standard observation instrument remains unchanged. The light intensity $I$ through the smoke into of the instrument or the naked eye, the observations result is light and dark, only when $\frac{1}{I_0} > 1 - \mu$, the observations bright. $\mu$ is the sensitivity of the instrument or the naked eye, $\mu$ is smaller, the instruments or the naked eye is more sensitive, usually $\mu << 1$.

A. Change Law of Smoke Concentration

The explosion occurred time is counted as $t = 0$, the coordinate points for the origin. Smoke concentration on time $t$ at any point $(x, y, z)$ in infinite space is $C(x, y, z, t)$. The assumptions of the unit time through unit normal traffic to the area as following

$$ q = -k \cdot \text{grad}C $$

K as Diffusion coefficient, grad as Gradient, $C$ as concentration. A negative sign indicates diffusion concentration by high concentration to low. $\Omega$ as spatial domain investigated, $V$ as volume, $S$ was surrounded by the surface, outer normal vector of $S$ is $n$, the flowing through $\Omega$ in the $[t,t+\Delta t]$ as following

$$ Q_1 = \int_{\Omega}^{t+\Delta t} \int_{S} q \cdot n \, d\sigma \, dt $$

The smoke increments within $\Omega$ as following

$$ Q_2 = \int_{\Omega} |C(x, y, z, t) - C(x, y, z, t + \Delta t)| \, dV $$

According to The law of conservation of mass

$$ Q_1 = Q_2 $$

And the Albright formula based on surface integrals

$$ \int_{S} (q \cdot n) \, d\sigma = \int_{V} (\text{div}q) \, dV $$

where div is the divergence of the mark, by (1) - (5), and then using the integral mean value theorem, we get

$$ \frac{\partial C}{\partial t} = k \text{div}(\text{grad}C) = k \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \right) $$

This is parabolic partial differential equations. Based on the assumption (a), the initial conditions is point source function in the origin, can be counted as

$$ C(x, y, z, 0) = Q \delta(x, y, z) $$

$Q$ as the total cast smoke, $\delta(x, y, z)$ as the point source function. The solution of the equation (6) satisfies the conditions (7) is

$$ C(x, y, z, t) = \frac{Q}{(4\pi kt)^{3/2}} e^{-\frac{x^2+y^2+z^2}{4kt}} $$

This result indicates that, for any moment, the smoke concentration surface $C$ is the spherical as $x^2 + y^2 + z^2 = R^2$, and with the increase of spherical radius $R$, the value of $C$ is reduced continuously.

B. Light Intensity Variation Through of Smoke

The light pass through the smoke in a certain direction, then the length coordinate on the direction account as $l$, $C(l)$ is the smoke concentration, $I(l)$ is the light intensity, in accordance with the assumptions (c), we will get

$$ \frac{dl}{dl} = -\alpha C(l)I(l) $$

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\( \alpha \) is light absorption coefficient, if the intensity of the light does not enter the smoke \((l = l_0)\) meter for \( I_0 \)

\[ I(l) = I_0 \] 

(10)

Solutions of Equation (9) under the conditions (10) is

\[ I(l) = l_0 e^{-\frac{\int_0^t C(s)ds}{4\pi \alpha}} \] 

(11)

C. Instrument (Naked Eye) Sensitivity and the Opacity Area Boundary

From the above analysis we can get smoke concentration in the space is changing continuously, the light intensity through the smoke into the instrument is changing continuously too, then the reason will be observed to the spread of smoke opacity when the boundary of the area has a larger first, then eventually disappears completely because, the instrument observations only light and dark of the points, while the bright and dark dividing line is determined by the sensitivity, based on the assumption that (d) only if the

\[ \frac{1}{l_0} > 1-\mu \] 

(12)

Observations are bright, and then you can think that the smoke has completely disappeared, taken along the axis of light, set viewpoint in place \( z = \infty \) and the light source in place \( z = -\infty \), when smoke completely disappeared, the condition (12) by the formula (11) can be written

\[ e^{-\alpha \int_{-\infty}^{\infty} C(x,y,z,t)dz} > 1-\mu \] 

(13)

Because surface \( C(x,y,z,t) \) is spherical, therefore, the area boundary of the spread of smoke projection on \( xy \) plane is the circumference, referred to as

\[ x^2 + y^2 = r^2 \] 

(14)

The variation \( r(t) \) over time of circumference radius is determined by the condition (13).

D. Smoke Diffusion and Dissipation Variation

Using approximate relations \( \ln(1+x) \approx x(x<<1) \), (13) can be turned into

\[ \int_{-\infty}^{\infty} C(x,y,z,t)dz < \frac{1}{\alpha} \ln \frac{1}{1-\mu} = \frac{\mu}{\alpha} \] 

(15)

The formula (8) is integrated by substituting into (15), and using the formula \( \int_{-\infty}^{\infty} e^{x^2}dx = \sqrt{\pi} \alpha \), we will get

\[ \frac{Q}{4\pi kt} e^{-\frac{x^2+y^2}{4\alpha t}} = \frac{1}{\alpha} \] 

(16)

Smoke spread radius of the area can be impressed by

\[ r(t) = \sqrt{\frac{4k}{\ln \left( \frac{\alpha Q}{4\pi kt} \right) - \mu}} \] 

(17)

Formula (17) can be drawn in Figure 1, when

\[ t = t_1 = \frac{aQ}{4\pi ke} \] 

(18)

Smoke diffusion region radius \( r \) reaches its maximum \( r_m \), when

\[ t = t_2 = \frac{aQ}{4r_k} \] 

(19)

\( r = 0 \), at the moment \( t_1 \), smoke disappear completely. \( (18), (19) \) shows that: \( t_1, t_2 \) is proportional to the light absorption coefficient \( \alpha \) and Smoke who discharge \( Q \); inversely proportional to the diffusion coefficient \( k \), from (18), (19), we can get \( t_2 = t_1 \cdot e \), So when smoke diffusion region after the maximum moment \( t_1 \), it can be predicted moments \( t_2 \) when smoke disappear completely.

![Figure 1](image)

We get the mathematical model of smoke diffusion and dissipation of shells’ explosion; it has two advantages as following:

(1) The smoke diffusion and dissipation is obeyed to the physical rule. The results obtained only from the physical rule can not explain the process of diffusion and dissipation of smoke we observed well, we introduces an instrument indicator or sensitivity of human eye; it can explain the phenomenon above successfully and get the correct conclusions.

(2) The factors related to the smoke diffusion and dissipation has been analyzed, the light intensity variation has been researched, and the precise time of the smoke disappearing is forecast. The model has laid the foundation for the data visualization in the virtual reality platform.

E. Particle System Model

Particle system model is the most successful way to simulate dynamic irregular fuzzy objects. The basic idea is to use a large number of vitality particle elements to describe the irregular nature fuzzy scene. Particles are objects that have mass, position, and velocity, and respond to forces, but that have no spatial extent. Because they are simple, particles are by far the easiest objects to
simulate. Despite their simplicity, particles can be made to exhibit a wide range of interesting behavior. For example, a wide variety of non rigid structure can be built by connecting particles with simple damped springs. In this portion of the course we cover the basics of particle dynamics, with an emphasis on the requirements of interactive simulation. The properties like position, velocity, color, size, and transparency of the particles changes follow certain principles. The movement of particles is controlled by combing particle system and mathematical model can overcome randomness shortcomings of the particles themselves.

(a) The generally simulating process of particle system

(1) The basic steps to render smoke each frame according to the general procedure of particle system as following:

(2) A new smoke particle and field of force are generated;

Each one new smoke particle and field of force is given certain initial properties;

(3) The smoke particles which are over the lifetime is removed;

(4) Updating the properties of field of force, such as position and velocity according to the rules that is first set;

(5) Updating the color, size, transparency and other properties of particles according to the rule; update the position and speed properties of particles by the smoke force;

(6) Detecting the collision with outside world, and if collision occurs, the collision is eliminated;

(7) Rendering the particle to generate image.

We can get a dynamic loop process by going step (3) to (7) continuously.

(b) Our algorithm processes of Exploding

Since the explosion happened in an instant time, when the explosion occurred later no longer produce new particles, so the particle emission type can use explosive particle system model, that is, at the same time all particles launched from a region and outward spherical expansion. At beginning, the smoke expansion rapidly, then gradually slowed down until disappearing. The whole process can be simplified as following, the initial position of the smoke particles is randomly generated near the origin, the explosion smoke outward diffusion by sphere radius, and change with time to a maximum radius of the explosion, the particles initial speed is larger, and changes over time gradually until decrease; the particle size gradually increases over time. The phenomenon of fog rolling is occur when explosion, in order to simulate this phenomenon, the explosion is initially fast rotating particle texture, and then reduce the rotational speed varies with time. The same time the color of the particles need be set, and the rate of change in transparency, and gradually adjust the color and transparency of the particles in the particle movement process, when the particles extended to a maximum radius, the particle color gradually weakened, and fusion with the background, smoke disappear. The algorithm processes is shown in Figure 2.

III. RENDERING OF SHELLS EXPLODING

The render of the smoke is a very important part. Particle-based simulation of smoke can be used OpenGL [6], and texture mapping can be used to improve realistic. When rendering the smoke particles, we can use the Billboard technology; it is a two-dimensional texture mapping technology [7].

A. Texture Mapping

In computer graphics, the application of a type of surface to a 3D image, a texture can be uniform, such as a brick wall, or irregular, such as wood grain or marble. The common method is to create a 2D bitmapped image of the texture, called a “texture map,” which is then “wrapped around” the 3D object. An alternate method is to compute the texture entirely via mathematics instead of bitmaps. The latter method is not widely used, but can create more precise textures especially if there is great depth to the objects being textured.

A texture map is applied (mapped) to the surface of a shape or polygon. This process is akin to applying patterned paper to a plain white box. Every vertex in a polygon is assigned a texture coordinate (which in the 2D case is also known as a UV coordinate) either via explicit assignment or by procedural definition. Image sampling locations are then interpolated across the face of a polygon to produce a visual result that seems to have more richness than could otherwise be achieved with a limited number of polygons. Multi texturing is the use of more than one texture at a time on a polygon. For instance, a light map texture may be used to light a surface as an alternative to recalculating that lighting every time the surface is rendered. Another multi texture technique is bump mapping, which allows a texture to directly control the facing direction of a surface for the purposes of its lighting calculations; it can give a very good appearance of a complex surface, such as tree bark or rough concrete that takes on lighting detail in addition to the usual detailed coloring. Bump mapping has become popular in recent video games as graphics
hardware has become powerful enough to accommodate it in real-time.

B. The Representation of Particle Texture Image

Particle texture image can be generated by the algorithm or an existing image. The texture is described by using RGBA. And R = G = B = A. A is transparent, transparency is smaller when gray is bigger. Because the thickness of the particles descending from the center to the edge, it can be considered the gray values of texture also obey to the rule, while it is required c transition continuously. Generally this trend can be use Gaussian distribution formula to distribution, as following:

\[
h(d) = \frac{\rho}{2\pi^{3/2}\sigma} \exp\left(-\frac{d^2}{2\sigma^2}\right)
\]

(20)

where: d represents the length from the sphere center to edge; h (d) represents the gray value of texture from the sphere center to distance d; the variance \( \sigma \) is the Gaussian distribution variance, in order to achieve normalization (i.e. texture data is only defined in the \([-1.0, 1.0]\)), \( \sigma \) will be taken as 3; \( \rho \) as the modulation values of central peak, the it can adjust the maximum gradation;

P is in the range \([0, 2\pi^{3/2}\sigma]\), we found the peak is taken as 0.4, the simulation results would be more appropriate.

The Particle texture image produced by Gaussian distribution formula is shown in figure 3.

C. Billboard Technology

The basic idea of the Billboard is using two-dimensional texture instead of the more complex three-dimensional geometric entities to improve the realism efficiency [8]. The essence of the technique of the Billboard is the plane rotating always facing the observer’s viewpoint. Texture patterns are mapped by texture, affixed to the surface of the polygon on the calculated viewpoint and the relative position of the particles, and then the particles texture mapping plane is rotated, it is always perpendicular to the view vector; whether viewpoint how translational rotation, seen from the perspective of the observer, are real three-dimensional space objects, eliminating the distortion due to viewpoint transformation.

BillBoard realization lies in two places, one is the mode of texture mapping, for rendering irregularly fuzzy objects, usually with transparent textures mode, where we can use Alpha test function in OpenGL. Particle systems are generally used for the Alpha Blending function. The second key is how to make the a polygon face always toward the viewer, when rendering the particles, mainly for changing viewer cones, adjusting the normal vector of each particle in data field in dynamically, so that it is on the cone midline, thereby maintaining consistent rendering. Basic principles as follows:

Since our goal is to make the polygon maximize \( t \) to the final scene, so simplicity, we need to adjust the position of view vector coincides with the normal vector, as shown in Figure 4. Because the normal vector ultimately reflected by the coordinates of the vertices, so, as long as the four unit vectors (upleft, bottomleft, upright, bottomright) are calculated. Specific implementation steps are as follows.

1. \( \hat{v} = (x \ y \ z) - (eyex \ eyey \ eyez) \); 
2. \( left = v \times up \);
3. Because \( up \) is not exactly perpendicular to \( v' \), it needs to be corrected \( up = v \times left \)
4. \( upleft = left + up \)
5. \( bottomleft = left - up \)
6. \( upright = up - left \)
7. \( bottomright = -upleft \)
8. Finally obtained four vertices coordinates of quadrilateral as follows:

\[
\begin{align*}
(x + upleft.x \ & \ y + upleft.y \ & \ z + upleft.z) \\
(x + bottomleft.x \ & \ y + bottomleft.y \ & \ z + bottomleft.z) \\
(x + bottomright.x \ & \ y + bottomright.y \ & \ z + bottomright.z) \\
(x + upright.x \ & \ y + upright.y \ & \ z + upright.z)
\end{align*}
\]

Billboard is a low-level, hardware-independent technology. It can be used for solving any questions about the normal vector angle of elements changing with viewer. When rendering we have adopted following approaches: using the depth buffer, rendering quadrilateral and texture under depth buffer, and then use transparent, mixed effects and linear texture filtering can significantly improve the display effect of smoke particles.

Finally, it’s worth noting that there are many different algorithms for blending colors in computer graphics. These are often referred to as “blend modes.” By default, when we draw something on top of something else in Processing, we only see the top layer—this is commonly referred to as a “normal” blend mode. When the pixels
have alpha transparency (as they do in the smoke example). Processing uses an alpha compositing algorithm that combines a percentage of the background pixels with the new foreground pixels based on the alpha values.

However, it’s possible to draw using other blend modes, and a much loved blend mode for particle systems is “additive.” Additive blending is in fact one of the simplest blend algorithms and involves adding the pixel values of one layer to another (capping all values at 255 of course). This results in a space-age glow effect due to the colors getting brighter and brighter with more layers.

IV. CONCLUSIONS

Finally, we develop a demo system based on our algorithm and show the results generated by our simulation algorithm. The simulation algorithm of explosion effects is implemented by Visual C++6.0 and OpenGL graphics library programming. The operation system is Windows XP, Graphics card is NVIDIA NVS 3100M, memory capacity is 1G, RAM 2G. When particle number is 40000, the frame rate is 60FPS.

Experimental results show that our algorithm can produce real time simulation of explosion in consumer PC Platform and the effect of smoke looks realistic. The visual simulation result shown in Figure 5, from the simulation results, we can see the smoke behavior of explosion is accord with the true scene.

![Figure 5. The explosion process based on our algorithm](image)

This text summarized and analogized the modeling method of dynamic irregular fuzzy objects at home and abroad. With a detail introduction to the classification of the works as well as different kinds of methods employed in the field. The methods applied mainly include the particle system method, the mathematic and physics-based method and the texture based method. We studied mathematical model first, then using thought of particle systems to management the spread and dissipation of smoke. Combining the mathematical model and particle system can overcome the randomness of the particles. When rendering, with texture mapping technology to improve simulation efficiency. The results show that the proposed algorithm is simple and practical. The next step is to consider visual effects, including dissipation, whirlpool and obstacle collisions.

The simulation algorithm based on particle system has the following advantages: can produce more realistic images; The structure of the particles are simple to set up, it is easy to modify parameters; The drawing method is flexible and real-time. The dynamic image of the explosion is produced by using particle system series method, the set of values are the preliminary observations of the particles, and dynamic behavior of the particles is controlled according to the characteristics of the object itself, and mathematical dynamic equation.

Designing and developing a real-time and efficient, simulation method and the practical software has been the people’s important work, but is a difficult work. Using computer to simulate the natural phenomenon effectively, there are many problems for us to study deeply and in detail. The next step is needed to use light Tracing Ray technology to establish the illumination model when exploding, and simulate the interaction of the flame and ambient light.

REFERENCES

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