On the Splitting Algorithm Based on Multi-target Model for Image Segmentation

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Abstract—Against to the different regions of membership functions indicated image in the traditional image segmentation variational model, resulting segmentation is not clear, de-noising effect is not obvious problems, this paper proposes multi-target model for image segmentation and the splitting algorithm. The model uses a sparse regularization method to maintain the boundaries of segmented regions, to overcome the disadvantages of segmentation fuzzy boundaries resulting from total variation regularization. The algorithm uses the multi-scale geometric analysis tool to maintain the geometry of image, which multi-scale geometric transformations can be flexibly selected according to actual problems. Experimental results show that: the proposed model and the algorithm are feasible and effective.

Index Terms—Segmented Regions; Membership; Rapid Convergence; Multi-scale Geometric Transformations

I. INTRODUCTION

Image segmentation plays an important role in computer vision as the basis for understanding and recognition of image, but due to the complexity of the image, there is no any good general image segmentation algorithm which can solve all problems, but even so, there are still waging scholars research into the ranks of image segmentation. In recent decades of study, image segmentation has made great progress. In the conventional image segmentation algorithm, there is segmentation based on region color information or based on edge information, but the use of too little information is not able to achieve the desired segmentation [1-4]. The algorithm based on image segmentation which cleverly combines the regional color and edge information has recently become a hot research. The graph cut algorithm applying to the image is completed in 1989 by Greig et al. [4-6], and subsequent studies have used a binary image, but its importance has not been in a very long time until the beginning of this century, Boykov et al [7-9], defined the definition of graph cut algorithm applying to image map and determined weights, and proposed a more efficient algorithm for solving the maximum flow based on the maximum flow algorithm of augmenting path algorithm. Their pioneering work led to the hot of image segmentation once again, and the algorithm based on image segmentation is endless.

As one of the fundamental problems of image processing and computer vision, image segmentation is the basis of image understanding but also a key step in image analysis. In the past 20 years, the image segmentation based on variational method has become a hot research field [10]. The classic image segmentation of variational model is active contour (AC) model and the geodesic active contour (GAC) model [11-15]. The minimization energy functional of these two models is dependent on the boundary characteristics. Mumford and Shah made his famous Mumford Shah (MS) model through combining boundary features of image and gray value information within the region. Chan and Vese et al. used piecewise constant functions and smooth function to propose a simplified model of MS. A lot of work expands around these models after been made [16].

Currently, the segmentation problem of the two goals (foreground and background) has been well studied. However, due to the complexity of the actual image, multi-object segmentation is still a challenging problem. The multi-target image segmentation methods based on variational model can be divided into two categories: one is a hard segmentation, for example: the level set method (area does not overlap); another is a soft segmentation (also called fuzzy segmentation), for example: use membership functions which can be overlapping to represent the segmented regions. The results showed that: multi-target hard segmentation based on level set method limits the number of applications with real-time computing requirements since the many iterations of algorithm. In recent years, multi-target soft segmentation technique has attracted the attention of many scholars.

As well known, with the development of multi-scale geometric analysis, sparse regularization method has been widely applied to the inverse problems, such as de-noising, de-blurring, source separation, repairing of image, etc [17-19]. It proposed a new multi-objective model of image segmentation and corresponding rapidly dividing algorithm based on sparse regularization method. In the traditional sparse representation model, assuming the image in a particular field (for example: the wavelet domain, Curve wave field) is sparse, that most of the atom coefficient is zero, only a few large non-zero coefficients, and zero coefficients reveal the internal structure and the nature of the image attributes. However, due to the complexity and diversity of the image, the sparse assumptions of image itself in a specific domain can not well describe the image. In the proposed variational image segmentation model, the membership function indicates the different regions of image segmentation, and its structure is relatively simple to use.
multi-scale geometric dictionary to effectively represent sparse.

This paper made expand and innovative work mainly in the following areas:

(1) Against to the different regions of membership functions indicated image in the traditional image segmentation variational model, resulting segmentation is not clear, de-noising effect is not obvious problems, this paper proposes multi-target model for image segmentation and the splitting algorithm. The model uses a sparse regularization method to maintain the boundaries of segmented regions, to overcome the disadvantages of segmentation fuzzy boundaries resulting from total variation regularization. The algorithm uses the multi-scale geometric analysis tool to maintain the geometry of image, which multi-scale geometric transformations can be flexibly selected according to actual problems.

(2) To further validate the correctness and validity of the proposed splitting algorithm, it makes simulation experiments of the TV regularization algorithms and level set algorithm. On synthetic image segmentation experiments, it greatly extracts marked target area, and ultimately gives a good segmentation results. In the comparison of running time and the number of iterations, the convergence speeds. In the multi-target segmentation experiment of brain’s MRI images, it successfully extracted the different categories of targets. The simulation results show that: the proposed algorithm has the advantages of simple, easy to implement, run fast, clear image, de-noising effect is obvious in image segmentation.

II. MULTI-TARGET MODEL FOR IMAGE SEGMENTATION

Given the original image \( t(y) : \Omega \rightarrow D \{0 \} \), assumed the image segmentation region number \( M \) is known. Potts made the following multi-target image model of hard segmentation:

\[
\min_{d \in \Omega, F \subseteq \Omega} \left\{ \sum_{i=1}^{M} |\Omega| + \lambda \sum_{i=1}^{M} \int_{\Omega} t_i(y) dy \right\}
\]

(1)

Here, the first term is the regularizarion term, the second term is the fidelity term (data items), \( \lambda \) is the regularization parameter which plays an important balance role between the regular items and fidelity term. In regular items, \( |\Omega| \) represents the side length of divided sub-regions \( \Omega \). In fidelity entry, one popular choice of \( t_i \) is \( |f(x) - m_i|^2 \), where \( m_i \) represents the sub-regional mean gray of \( \Omega_i \). Solving an effective numerical algorithm of the model is still a problem to be solved. By introducing the membership function \( l_i(x) (i = 1, 2, ..., M) \), \( l, F \), et al. relaxed Potts model and proposed the following multi-target image model for soft segmentation:

\[
\min_{0 \leq \alpha \leq 1} \left\{ \sum_{i=1}^{M} \int_{\Omega} |l_i(y)| dy + \frac{\lambda}{2} \sum_{i=1}^{M} \left[ \alpha \left( \sum_{j=1}^{M} l_j(y) - 1 \right)^2 \right] dy \right\}
\]

(2)

Here, the first term is based on the total variation (TV) regularization term, the second term is the fidelity term (data entry), and the third is a quadratic penalty term, whose role is to ensure the normalization constraint.

\[
\sum_{i=1}^{M} l_i(y) \neq 1
\]

(3)

The model can be solved by using the dual algorithm Chambolle, referred to herein as a TV regular algorithm. Model (2) has two main disadvantages: (1) Based on regular TV, it likely to causes blurred and shifting boundaries; (2) To ensure the normalization constraint, the parameter \( \eta \) must take big enough which will lead to the stability problems of calculating.

The membership function \( l_i(y) \in [0, 1] \), it indicates the segmentation sub-region of image \( \Omega \). Its structure is relatively simple, using multi-scale geometric dictionary sparse can effectively represent membership function. Based on this idea, it propose the following multi-target model of image segmentation.

\[
\min_{0 \leq \alpha \leq 1} \left\{ \sum_{i=1}^{M} \sum_{j=1}^{M} (\xi_i) + \lambda \sum_{i=1}^{M} \int_{\Omega} t_i(y) l_i(y) dy \right\}
\]

(4)

Among them, the first term is the regularization term and the second term is the fidelity term. In the regular entry, \( l_i(y) \) represents the multi-scale geometric transformations (wavelets, curve wave) decomposition coefficients of membership function \( l_i(y) \). \( t \) Indicates the index set of decomposition coefficients. In a data item, \( \lambda \) is the regularization parameter. It regulates the balance of the both.

\[
t_i(y) = -\log s_i(y), s_i(y) = \frac{1}{|\Omega|} \int_{\Omega} j_\nu(f(y) - f(x)) dy
\]

(5)

where \( j_\nu \) is a Gaussian kernel with the standard deviation \( u \), \( |\Omega| \) represents the area of divided sub-regions, Where \( f(y), f(x) \) expressed the original image which defined a bounded domain \( \Omega \). Other forms of data items can also be discussed in the framework of this article.

The main advantages of the new model are: (1) Under sparse constraint, it can better maintain the boundaries of segmented regions to overcome the disadvantages of too smooth boundary segmentation resulted by total variation regularization; (2) The use of multi-scale geometric transformation can better maintain the geometry of the divided regions. This is also the key point of which our model is better than TV regularization model. (Note: The dictionary used here is resolved, for example wavelet
dictionaries, curve wave dictionaries. In this framework, it can also use learning dictionary. Taking the computing speed and the timeliness of practical application into account, this article only discusses the sparse regularization constraint with using of analytic dictionary.)

III. SPLITTING ALGORITHM

By the model (3), by using the penalty function method, by increasing the quadratic penalty term, to obtain the following model.

\[
\min_{l} \left\{ \sum_{i=1}^{M} \sum_{y=0}^{N} |l_i|_1 + \lambda \sum_{i=1}^{M} \alpha l_i(y) l_i(y) \right\}
\]

\[
+ \frac{\mu}{2} \sum_{i=1}^{M} l_i(y)-1)^2 by
\]

\[
d.a.t.0 \leq l_i(x) \leq 1, i = 1, 2, ..., M
\]

This is the approximation of model (3), in order to meet the constraints I in model (3), \( \mu \) must take big enough, it will cause the stability problems of calculating. To solve this problem, based on the splitting idea of Bregman, it gives a fast splitting algorithm below. Introducing an auxiliary function \( l_i(y), i = 1, 2, ..., M \), through adding identity constraints \( t.0 = l_i(y), i = 1, 2, ..., M \), the model (3) is equivalent to the following model:

\[
\min_{l, \lambda} \left\{ \sum_{i=1}^{M} \sum_{y=0}^{N} |l_i|_1 + \lambda \sum_{i=1}^{M} \alpha l_i(y) l_i(y) \right\}
\]

\[
d.a.t.0 \leq l_i(y) = 1, i.t.0 \leq l_i(y) \leq 1
\]

\[
d.a.t.0 \leq l_i(x) \leq 1, i = 1, 2, ..., M
\]

where, \( t_i(y) \) represents the multi-scale geometric transformation decomposition coefficients of helper \( t_i \). By constraints III, it can see the model (6) is equivalent to the model (3). In the model (6), in order to satisfy the constraints \( t \), we use the projection formulas \( l_i = \min \{ \max \{ l_i, 0 \}, 1 \} \) in the iterative algorithm to project the membership function to the interval \([0, 1]\). To meet the constraints I, III, it increases quadratic penalty term, and introduces functions \( t_i(y) \) and \( l_i(y), i = 1, 2, ..., M \) to update iterative process. In order to write simple, the following iterative formula omitted variable \( x \), the final iterative formula of solution model (6) is as formula (7).

\[
\begin{align*}
\{ l_i^{m+1} \}_{i=1, 2, ..., M} &= \arg \min_{l_i} \left\{ \lambda \sum_{i=1}^{M} \alpha l_i(y) l_i(y) + \frac{\mu}{2} \sum_{i=1}^{M} l_i(y)-1)^2 by \right\}, \\
l_i^{m+1} &= \min \{ \max \{ l_i^{m+1}, 0 \}, 1 \}, i = 1, 2, ..., M
\end{align*}
\]

where, the parameters \( \mu \) and \( \eta \) are the penalty parameter, which introduced by the functions \( t_i(y) \) and \( l_i(y) \), to avoid the numerical instability caused by the excessive penalty parameter. The convergence solution of iteration system (8) is equivalent to the solution of model (1), it is the solution of model (2). In the below, it will discuss how to solve the sub-optimization problem of (2). The first optimization problem of iterative system (8) is a differentiable optimization problem, its Eu-ler-Lagrange equation about \( \lambda_i \) is:

\[
\lambda_i + \mu \left( t_i - t_i^n + t_i^n \right) + \eta \left( \sum_{i=1}^{M} l_i + t_i^n \right) = 0, i = 1, 2, ..., M
\]

From the formula (9), it gives that:

\[
l_i = \frac{-\lambda_i + \mu \left( t_i - t_i^n + \lambda_1 - s_i^n \right)}{\mu} = \frac{\lambda_i - \lambda_i + \mu \left( t_i - t_i^n + \lambda_1 - s_i^n \right)}{\mu + M \lambda}
\]

The second optimization problem of iterative system (7) can be decoupled as:

\[
u_{i}^{m+1} = \arg \min_{\nu} \left\{ \sum_{i=1}^{M} l_i^{m+1} + \frac{\mu}{2} \sum_{i=1}^{M} l_i(y)-1)^2 by \right\},
\]

\[
i = 1, 2, ..., M
\]

Use the symbolic with the subscript \( s \) to represent the function decomposition coefficients of multi-scale transform. By using the Parseval equation of multi-scale transformation, formula (12) can be written as:

\[
\{ u_{i}^{m+1} \} \lambda \in s = \arg \min_{\lambda \in s} \left\{ \left( l_i - u_i^n - t_i^n \right)^2 by \right\},
\]

\[
i = 1, 2, ..., M
\]

Therefore, for all indicators \( \lambda \in s \), the above formula could be further decoupling as:

\[
\{ u_{i}^{m+1} \} \lambda \in s = \arg \min_{\lambda \in s} \left\{ \left( l_i - u_i^n - t_i^n \right)^2 by \right\},
\]

\[
\forall \lambda \in s, i = 1, 2, ..., M
\]

Formula (15) is a classic \( l \) optimization problems, the solution of formula (15) can be obtained by soft threshold value. Hutchison soft threshold operator is \( l_i \), namely \( l_i | \alpha| = sign(\alpha) \left| \frac{\alpha}{|\alpha| - 1} \right| \). Therefore, for all indicators \( \lambda \in s, i = 1, 2, ..., M \), the solution of formula (15) is:
\[ \begin{bmatrix} u_{m+1} \\
\end{bmatrix} = S_{/u} \left( (l_{m+1}^\ast - l_m^\ast) \right) \]  

From the above analysis, the algorithm of solution to model (2) are summarized as below:

Algorithm 1 Splitting Algorithm

Input: Segmented image \( f \), parameters \( \lambda, s, u, \alpha \), multiscale geometric transform \( (l)_{low} \);

Initialization: \( t^0, u^0, l^0 \), if \( m = 0 \)

Iterated until meet \( \sum_{i=1}^{M} (l_{m+1}^i - l_m^i)^2 < \varepsilon \) or to achieve a fixed number of iterations

By type(5), calculation \( l, i = 1, 2, \ldots, M \);

d by displaying expressions (13) calculation \( l_{m+1}^i, i = 1, 2, \ldots, M \);

use of multiscale geometric transform coefficient \( (l_{m+1}^i, l_m^i) \). The use of the inverse transform to get

\[ u_{m+1}^i, i = 1, 2, \ldots, M \);

\[ t_{m+1}^i = l_{m+1}^i + u_{m+1}^i - u_m^i, i = 1, 2, \ldots, M \);

\[ t_m^i = \text{max} \left\{ \text{max} \{ (l_{m+1}^i, 0), l \}, i = 1, 2, \ldots, M \right\} ;

m = m + 1

Output: \( l, i = 1, 2, \ldots, M \).

Algorithm 1 Using split optimization techniques to make the model (3) become very simple to solve. From the point of view detailed description of the algorithm, the algorithm is essentially a special algorithm for iterative threshold. The main advantage of the proposed algorithm is as follows: (1) Because it gives the display expression of membership function and the algorithm does not need internal iterative, so it operates fast; (2) The algorithm is simple and easy to implement, thus ensuring rapid convergence of Bregman iteration; (3) The algorithm can flexible select multi-scale geometric transformation according to the image characteristics and needs of the application, for example, Wavelet, Curvelet, Bandelet, Contourlet, etc.

IV. EXPERIMENTAL SIMULATION AND ANALYSIS

A. Experimental Environment and Setup

To verify the feasibility and effectiveness of the new method, the paper makes simulation by carrying out a large number of test images. The experiment results is achieved under a CPU is Intel1.74GHz and a programming environment is Matlab 7.0.

In algorithm 1, the initializing of membership function \( l, i = 1, 2, \ldots, M \) needs to satisfy the conditions \( l \) and \( s \) of model(2). When the membership function that satisfies the constraints \( l, s \), they are overlapping each other. To simplify the initialization method, we use the initialization \( l^0(i = 1, 2, \ldots, M) \) which do not overlap each, and using different types of lines to mark in experiments and give a specific description. The initialization of membership function can also use clustering method and it can accelerate the convergence speed. In algorithm 1, the auxiliary variable \( u, s, b \) are respectively initialized as \( u_0^0, u_0^s, l_0^b \), \( l_0^b = 0, s = 0, b \) where, \( i = 1, 2, \ldots, M \). The standard deviation of the Gaussian kernel function \( \lambda = 1 \), the penalty parameter is Unified chosen as \( su = 11 \) and \( \eta = 10 \). The algorithm requires pre-set number of divided regions \( N \), the experiment and the actual characteristics of the image needs to set the value of \( N \). Another important parameter of the algorithm is the regularization parameter \( \lambda \), this parameter plays a role as balancing data items and regularization term. Experiments show that, \( \lambda \) selects [100, 400] can give a satisfactory segmentation results. Multi-scale image geometric transformation can select flexible according to different images characteristics, then uniformly select DB3 wavelet transform and 3-layer wavelet decomposition. It also gives the experimental results of \( s \) curve wavelet transform.

B. Analysis

The membership function in model (3) indicates different segmentation regions of image. Under sparse constraints, they can be used to mark the segmentation regions. To illustrate this point, it will use the proposed algorithm to do multi-target segmentation \( (N = 3) \) to synthetic images. Figure 1 (a) represents the membership function of original image and initialization, the thick lines mark as the, the thin lines mark as \( l_2^0, l_3^0 = 1 - l_0^0 - l_2^0 \). It can see the membership function \( l, i, l, \), optimized by the proposed algorithm (respectively as figure 1 (c), (d), (e)), the extraction well marked target area, the final figure 1 (b) segmentation results are given.

![Figure 1. Segmentation experiments for synthetic image of the proposed algorithm](image)

![Figure 2. The segmentation experiments for single-target noise image of the proposed algorithm](image)

![Figure 3. The segmentation experiments for dual target noise image of the proposed algorithm](image)
algorithm is still able to extract different goals. Experimental results show that the algorithm has the certain noise robustness.

Figure 4 shows the segmentation experiments of single-target image. It gives the proposed algorithm with using of wavelet and curves wave, the experimental results of TV regularization algorithm and the level set method. From the partially enlarged image of segmentation results, it can be seen that, the use of multi-scale geometric transformation can apart fingers, and the curve wave can better maintain the shape of the original image (Figure 4 (f) and (g)). Figure 4 (f) shows the results of level set method, and it clearly see that the two fingers can not be separated by the evolution curve. From figure 4 (h), it can be seen that, as with a regular TV boundary shifting, resulting the two fingers can not be apart by a regular TV algorithm.

Figure 5 shows the experimental natural image segmentation. Since this algorithm uses wavelet and wave curve segmentation results considerably, so here it only gives the experimental results of wavelet. Figure 5 (a) is the initial membership function of original image and marking. Figure 5 (b), (c), (d) are respectively the segmentation results of the proposed algorithm, TV regularization algorithms and level set algorithm. From figure 5(d), it can be seen that, the level set method fails with a concussion texture segmentation of this image, which is the level set method is difficult to overcome the shortcomings. Figure 5 (e) and (f) are respectively the membership function of the proposed algorithm and TV regularization algorithm. It can see the use of multi-scale transformation algorithm does not cause blurred boundaries, and TV regularization algorithm led to the blurring of the boundary.

Table 1 lists the comparison between test run time and number of iterations. Since this algorithm does not require internal iteration, and the convergence speed, then it can be seen from Table 1 that, the proposed wavelet algorithm costs the least time in all the experimental results.
To illustrate this algorithm has some practical value, we measured the image on the algorithm for testing. Figure 8 (a) is the measured image for cross-section of asphalt concrete pavement test blocks. Image segmentation results can be judged as an important basis for a skeletal density, that is the distinction stone mixed with other materials (stone chips, bitumen, additives). Figure 8 (b) shows the experimental results of the wavelet algorithm. It can see that most of the stone-targets are split out.

Finally, consider the model data items (RGB color space) in 3-D vector promotion, the algorithm can be applied directly to the color image segmentation. This promotion is direct and will not be described here. Figure 9 is the segmentation results of a color image and the proposed method gives a more accurate segmentation result.

These experiments demonstrate the effectiveness and benefits of the proposed algorithm. For most test images, in contrast to regular TV algorithms and level set algorithms, this algorithm runs faster and with more accurate segmentation results. This algorithm uses the curve wave energy to curve singularity better, keep the image geometry, but the operation speed is slower than the wavelet.

V. CONCLUSION

The multi-scale geometric transformations of the proposed multi-target model for image segmentation and splitting algorithm can be flexibly selected according to the actual problems. It will make further exploration and application especially to the specific characteristics of the image. The model and algorithms can be put into more thorough study and expansion. In the model, considering use the learning dictionary to sparse regular of membership function, that of using learning dictionary sparse represents segmented regions. In the algorithm, it can use the iterative regularization method and inverse scale space to further enhance the image segmentation results. Experimental verification shows the effectiveness and advantages of the algorithm.

### REFERENCES


