A Modulated Hybrid Filter Bank for Wide-Band Analog-to-Digital Converters

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Abstract—It is difficult to use a single analog-to-digital conversion (ADC) to satisfy the requirements for conversion of an ultra-wideband signal. A parallel architecture for high bandwidth ADC, named cosine modulated hybrid filter bank, is presented to address this problem. First, the proposed architecture shifts the input signal spectrum by means of mixers. The modulated signal is channelized into smaller frequency subband signals using identical lowpass analog filters. Then the subband signals are digitized through identical narrowband ADCs, respectively. Finally, the digitized signals are up-sampled, then filtered and combined to reconstruct the digital representation of the original wide-band input signal. The digital filters are designed to use the eigenfilter method based on total least squares error criterion. Since the sample-and-hold circuits needed are only identical narrowband baseband circuits, the simplicity of the system makes the design easier and cheaper. Several design examples are used to illustrate the performance of the proposed system.

Index Terms—Analog-To-Digital Conversion; Parallel Hybrid Filter Bank; Cosine; Reconstruction

I. INTRODUCTION

Physical signals such as audio and electromagnetic signal are analogue in nature. However, all of these signals need to undergo several processing stages in electronic systems such as transmitters and receivers and so on. With the popularity of large-scale integrated circuit, digital signal processing and wide application of digital computer, most of the signal processing task is usually accomplished in the digital domain. Hence, analog-to-digital converter (ADC) are inevitable and an essential block in many system [1]. More and more fields needs high-speed and high-precision ADC to sample ultra-wideband signal such as ultra-wideband radar signal processing system, software-defined radio of wireless communication and direct digital receiver. It is difficult to use single ADC to sample ultra-wideband analog signal. Parallel channel sampling based on generalized sampling theory is an effective method to achieve high-speed A/D conversion [2]. Many parallel structures are proposed to use for realizing high-speed sampling such as time-interleaved (TI) structure [3-9], quadrature mirror filter bank (QMFB) structure [10], hybrid filter bank (HFB) [11-18] structure, delta-sigma filter bank structure [19-21] and frequency conversion hybrid structure [1] [22]. The main idea of the parallel ADC structures is to use many channels to segment broadband analog input signal, and the channels use low-bandwidth ADC to sample and quantize the segmented signals, and the sub-band signals are combined to get the digital reconstruction of the original bandwidth analogue input signal. The advantage of the structure is that a paralleled high-speed and low-rate ADC can be used to acquire equivalent and high-sampling rate ADC.

TI structure, dating back to the early 1980s, is one of the parallel ADC structures which have been widely studied. In this architecture, parallel lower-speed ADC converters, are interleaved in time to result in an overall higher-bandwidth ADC. The ideal TI structure can improve the sampling rate of the system greatly, but the system is very sensitive to the gain, phase mismatch and clock mismatch between ADC, which greatly limits the conversion speed and resolution of the system in practice. Moreover, the implementation of TI structure needs high-speed sampling to hold circuit, which makes it difficult to implement circuit.

Petraglia and Mitra exploited QMFB and the concept of multirate signal processing, to introduce a new parallel ADC architecture based frequency-band decomposition. This parallel ADC structure based on QMFB overcomes the disadvantage of TI to some extent [10]. Parallel QMFB structure firstly uses the discretized analogue signal as the input signal of the system, and then uses discrete-time analysis filter based on switched capacitor to decompose analogue signal. The analysis filters consist of an array bandpass filters except for the first filter which is a low-pass filter. Also, the analysis filters are designed to divide the input signal approximately into equal spacing sub-channels. After the sub-band signal goes through the sampler and quantizer, digitally synthesized filter synthesizes them to get output signal. QMFB architecture is not as much sensitive to mismatch as TI structures [1]. This is basically due to the fact that the reconstruction filters can be designed by taking the
channel component mismatches into account. However, the disadvantage that parallel structure needs high-speed sampling to hold circuit is not solved. Moreover, the use of switched-capacitor filters limits the speed of the system and introduces switching noise, which may degrade the designed system performance. The HFB ADC structure is similar to QMFB in principle, and the difference between them is that HFB input is analogue signal, and the analysis filter applies analog filter without using switch capacitor circuit \[11-13\]. Therefore, the HFB inherits all the advantages of QMFB, including less sensitivity to channel mismatch and clock mismatch compared to the TI. Moreover, because the HFB does not use the switched capacitor circuit, switching noise cannot be introduced. Hence HFB improves the resolution of the sampling system greatly. However, HFB doesn’t convert signal high-frequency band into baseband for processing. The system still needs high-speed sampling to hold circuit. Moreover, in order to segment broadband input, HFB system needs to design band-pass filter with high center frequency, and therefore implementing circuit is difficult.

Delta-sigma modulation can be exploited together with the above mentioned parallel architecture, namely the TI structure or the QMFB structure, to result in a high-bandwidth, high-resolution system \[19\]. However, the bandwidth of such systems in very limited in fact \[1\]. Frequency-domain transformation hybrid structure proposed in literature \[1\] \[22\] uses orthogonal baseband transformation technology to convert sub-band signal into baseband for processing, which solves the disadvantage caused by high-speed sampling holding circuit, but the structure introduces orthogonal mismatch error, which limits the resolution of the system. Compared with TI, QMFB and HFB structure, the structure in literature \[22\] needs twice as much as ADC, which makes the implementation of system more difficult.

The main work in this paper is twofold. First, the paper proposes a new parallel structure to realize high-speed and high-precision ADC which is called parallel modulation hybrid filter bank ADC structure. In this architecture, a wideband analog input signal is frequency-translated using cosine modulators down to baseband and lowpass filtered before being converted into digital by subband ADC. After conversion into digital, the digitized signals are up-sampled by up-samplers and reconstructed by digital synthesis filters. A similar structure is proposed in literature \[23\]. However, the modulation system of the modulator proposed in literature \[23\] for input signal only can be applied to two channels. When the number of channels is greater than two, after the input signal is modulated by the modulate system in literature \[23\], the low-pass filter cannot capture the information of input signal spectrum. Hence, the system in literature \[23\] cannot reconstruct input signal. The structure proposed in the paper is suitable for any \( M \) channel \((M \geq 2)\). Second, when the modulated HFB is designed, given the cosine modulators and analog lowpass filter, the finite impulse response (FIR) digital synthesis filters are designed using the total least squares eigenfilter method. The coefficients of FIR filters are obtained by finding the eigenvector corresponding to the minimum eigenvalue of a related real, symmetric and positive definite matrix. The simulation results prove that the structure proposed in the paper is effective.

II. MODULATION HYBRID FILTER BANK ADC STRUCTURE

A. Structure Description

![Figure 1. Modulated hybrid filter bank for ADC](image)

The parallel hybrid filter bank ADC structure proposed in the paper is a modulation structure, as shown in Figure 1. While simulating input signal \( x(t) \), wideband goes through \( M \) channels \((M \geq 2)\). Each channel includes an analog multiplier, low-pass analog analysis filter \( H(s) \), ADC, sampler \( \uparrow M \) and digitally synthesized filter \( F_m(z) \), \( m = 0, 1, \ldots, M-1 \). And the synthesized filter synthesizes each sub-band signal into the final output signal \( y(n) \).

To simplify the description, the \( m \) channel is taken as an example for instruction. It is supposed that the angular frequency spectrum \( X(\omega) \) band limit of wideband analog input signal \( x(t) \) is \([(-\pi/T), (\pi/T)]\) in which \( T \) is the overall sampling period of the system. If the sampling rate of \( 1/T \) is used to sample signal \( x(t) \), the signal spectrum has no aliasing. For ultra wideband signal, the sampling rate of single ADC is difficult to reach \( 1/T \), so it is unable to sample by using the rate of \( 1/T \).

The signal \( x(t) \) firstly uses the multiplying unit to multiply \( \cos \omega_m t \) for modulation, so the system is modulation system. After modulation, \( X(\omega) \) moves \( \pm \omega_m \), \( \tilde{X}_m(\omega) = \pi[X(\omega + \omega_m) + X(\omega - \omega_m)] \), in which \( \omega_m = \pi m/(MT) \). \( \tilde{X}_m(\omega) \) means the frequency spectrum of \( x_m(t) \). The pass-band cut-off frequency of low-pass filter \( H(s) \) is

\[
\omega_m = \pi/(MT) \tag{1}
\]
After the input signal spectrum \( X(\omega) \) is segmented into \( M \) sections averagely, the \( m \) section is taken out without considering the influence of quantizing noise. The sampling frequency of ADC is \( \frac{1}{(MT)} \). The signal which is sampled by ADC needs the designed digital filter \( F_n(z) \). \( F_n(z) \) needs to make the signal introduced by ADC sample distorted and make aliasing error minimal. Therefore, the overall effective sampling rate of the system corresponds to sampling rate of Nyquist. Because the analysis filter of the system is analog filter and synthesized filter is digital filter, the system is a hybrid filter bank system.

\[
X(\omega) \quad \text{after passing the second route of processing and moving } \pm \pi/2 \text{ (it is represented by } X_{10}(j\omega) \text{ and } X_{11}(j\omega)).
\]

\( X_{10}(j\omega) + X_{11}(j\omega) \) means the frequency spectrum of input signal after modulation and low-pass filtering. The positive partial band of two routes of sub-band signal includes the information of 1 and 2, and the negative partial band includes the information of 1’ and 2’, so two routes of sub-band signal includes all information of \( x(t) \). Designing synthesized filter bank can get sampling output \( y(n) \) corresponding to \( x(t) \).

The system has the following advantages: (1) As sampling is made after modulating input signal and low-pass filtering or after wideband signal is decomposed into narrow sub-band baseband signal, the required sampling hold circuit can be implemented by the equivalent narrow-band baseband, which solves the problem that the structures in [1] [3] [10-11] need high-speed sampling hold circuit. (2) The system only needs to design low-pass analog filter, which solves the difficulty that the structure in literature [11] needs to design band-pass filter with high-center frequency. (3) The number of ADC required by the system is equal to the number of channels which reduces by half of the number of ADC for the structure in literature [22], which reduces the complicity of system implementation. (4) The angular frequency modulation of the system modulator is \( \omega_n = \pi n (MT) \). Compared with the modulation mode in literature [23] which is only applied for two channels, the modulation mode in the paper is appropriate for any limited channel.

**B. Synthesized Filter Design**

In figure 1, after modulating, filtering and ADC sampling the wideband analog signal \( x(t) \), the frequency spectrum of the achieved sub-band signal \( x_n(n) \) of the first route is written as

\[
X_0(e^{j\omega}) = \frac{1}{MT} \sum_{n=-\infty}^{\infty} X\left(\omega - \frac{2\pi n}{MT}\right) H\left(\omega + \frac{2\pi n}{MT}\right)
\]

The frequency spectrum of the achieved sub-band signal \( x_n(n), m = \{1, \ldots, M-1\} \) is written as

\[
X_n(e^{j\omega}) = \frac{\pi}{MT} \sum_{n=-\infty}^{\infty} X\left(\omega - \frac{2\pi n}{MT}\right) H\left(\omega + \frac{2\pi n}{MT}\right) + \cdots
\]

\[
X_m\left(\frac{\omega}{M} + \frac{j2\pi n}{MT}\right) \quad m - 2n \rightarrow n \quad \text{is substituted into formula (2), which can get}
\]

\[
X_0(e^{j\omega}) = \frac{1}{MT} \sum_{n=-\infty}^{\infty} X\left(\frac{\omega}{M} + \frac{j\pi n}{MT}\right) H\left(\frac{\omega}{M} + \frac{j\pi n}{MT}\right)
\]

Substituting it into formula (3) can get

---

**Figure 2.** Modulating and filtering the input signal for two-channel case.
can be represented as 

\[ X_n(e^{j\omega}) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kMT} + j\frac{n\pi}{MT} + j\alpha_n \ldots \]

(5)

\[ H\left(\omega = \frac{n\pi}{MT}\right) \]

From formula (4) and formula (5), we can see that the above variable substitution is to convert the modulation

\[ Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} X_n e^{j2\pi kMT} + j\frac{n\pi}{MT} + j\alpha_n \ldots \]

(6)

Substituting formula (4) and (5) into formula (6) can get formula (7) (as shown at the bottom of the paper). The spectral band limit of signal \( x(t) \) is \([ (-\pi T), (\pi T) \] , so the sum term about \( n \) in formula (7) can be limited to be \( n = -(M-1), \ldots, M - 1 \). The paper only considers that the input signal spectrum is \([ 0, \pi T ] \) and \( n = 0, 1, \ldots, M - 1 \). The ideal synthesized

\[ T_n(e^{j\omega}) = \frac{1}{MT} H\left(\omega = \frac{n\pi}{MT}\right) F_n(e^{j\omega}) + \frac{\pi}{MT} \sum_{n=-\infty}^{\infty} H\left(\omega = \frac{n\pi}{MT} + j\alpha_n\right) \]

(7)

where \( d \) is an arithmetic number, which is system delay. The expression of \( T_n(\omega) \) is shown in formula (9) (as shown at the bottom of the paper). It is general that \( T_n(\omega) \) is a distortion function which means the gain and phase response of the system. \( T_n(\omega) \), \( n \in \{1, \ldots, M - 1\} \) is called aliasing error function which means aliasing error generated by the system.

When the system is completely reconstructed, which meets the formula (8), formula (9) needs to satisfy the following condition.

\[ T_0(e^{j\omega}) = e^{-j\omega d}, \quad T_n(e^{j\omega}) = 0, \quad n \in \{1, \ldots, M - 1\} \]

(10)

It is supposed that the length of synthesized filter is limited FIR filter with the length of \( N \). The frequency response of synthesized filter is

\[ F_m(e^{j\omega}) = \sum_{k=0}^{N-1} f_m(n) e^{-j2\pi kM} = e(\omega) f_m \]

(11)

where \( f = [f_0^T, f_1^T] \) is the coefficient vector of synthesized filter bank, \( t(\omega) = [e^{-j\omega d}, 0]^T \). The expression of \( H_\omega(\omega) \) is shown at the top of the next page in the paper. And the design of synthesized filter is that the expected response \( t \) is given to solve the coefficient vector \( f \) of synthesized filter, which makes response \( H_\omega(\omega) f \) close to \( t \) as far as possible, which makes formula (12) possible.

Paralleling ADC synthesized filter ban coefficient vector under overall least square error criterion can be achieved by error function under minimization.

\[ J(f) = \int_0^\pi \frac{\|H_\omega f - f\|^2}{1 + jH_\omega f} d\omega = \frac{\hat{f}^H \hat{f}}{\hat{f}^H \hat{f}} \]

(13)

In the formula, \( \hat{f} = [f^H, -1]^H \), \( P \) is Hermitian matrix.

\[ P = \begin{bmatrix} A & b \\ b^H & g \end{bmatrix} \]

(14)

\[ A = \int_0^\pi H_\omega^H H_\omega d\omega \quad b = \int_0^\pi H_\omega^H t d\omega \quad g = \int_0^\pi t^H t d\omega \]

The subscript \( ^H \) represents the conjugate transpose of vector or matrix.

The coefficient vector \( f \) of conjugate filtered is constant, and \( P \) is Hermitian matrix, formula (13) can be simplified as
The synthesized filter can be written as follows:

\[ J(f) = \frac{\hat{f}^T \text{Re}(P) \hat{f}}{\hat{f}^T \hat{f}} \quad (15) \]

\[ H_e(\omega) = \begin{bmatrix} H(j\omega)e(\omega) \\ H(j\omega + j\pi/(2T))e(\omega) \end{bmatrix} \pi \left[ H(j\omega - j\pi/(2T)) + H(j\omega + j\pi/(2T)) \right]e(\omega) \]

\[ \min_f J(f) = \frac{\hat{f}^T \text{Re}(P) \hat{f}}{\hat{f}^T \hat{f}} \quad (16) \]

The symbol \( \text{Re}(\bullet) \) means the real part of complex value or complex vector. The solution of coefficient vector \( f \) of synthesized filter can be written as follows:

The overall aliasing error function is

\[ T_{\text{total}}(\omega) = \sqrt{\sum_{\omega} T_e(\text{Re}(\omega))^2} \quad (18) \]

The amplitude, phase distortion, hybrid aliasing error amplitude of the designed two-channel \((M = 2)\) modulating ADC system are shown in Figure 3. We can see from the figure that the overall aliasing amplitude of the system near zero frequency is great, and the distortion and overall aliasing amplitude near signal band edge is great, the overall aliasing error amplitude reduces to -24dB, and the distortion and aliasing amplitude also reaches -0.25dB. For most practical application, the reconstruction performance of the system is low. In order to improve the performance of the system reconstructing signal, the overall sampling frequency of the sampling system should be greater than Nyquist rate, which is a good solution [15]. The following simulation is to sample the input signal with the sampling rate of 4%. But the problems that the reconstruction performance of the system near zero frequency is bad and cannot be solved. Therefore, guard band in literature [18] is introduced. 3.5% of guard band is set at zero frequency, and the effective spectral range after the reconstruction of input signal sampling is \([0.035\pi, 0.96\pi]\). The other frequency bands are not concerned. Figure 4 shows the overall aliasing margin of error of the redesigned two-channel modulation ADC system. The distortion of the designed system amplitude is lower than \(2.8\times10^{-4}\)dB, and the phase distortion is lower than \(1.5\times10^{-5}\) rad, and the maximally overall aliasing margin of error is -89dB, which indicates that the performance of system reconstruction after setting guard band and sampling is improved greatly. The performance indicators are as shown in Table 1. In order to make the comparison easy, besides the difference between analog analysis filter and structure in literature [15], an eight-channel \((M = 8)\) modulation hybrid filter bank ADC system is designed. HFB system which is designed by using LMSGAD method in literature [15] under the condition that the sampling rate is 4% is compared with the system performance indicator designed in the paper, and the comparison results are shown in Table 2. We can see from table 2 that the maximal overall aliasing amplitude of the system designed in the paper is -79dB, which has no great difference from the maximal overall aliasing amplitude of the system in literature [15] which is -86dB. Similarly, the maximal amplitude distortion of the system designed in the paper is \(4.2\times10^{-4}\)dB, which is close to that of the system in literature [15]. We can see that the reconstruction performance of two systems is very similar. The analog of the system in literature [15] needs to design band-pass analog filter with high-center frequency, and the inaccuracy of the real circuit implementation can bring great error to the system. Although the order (four orders) of the analog filter of the system in the paper is higher than that of the system in literature [15] (two orders), the structure of the paper only needs to design the same low-pass analog filter, which greatly reduces the possibility that the inaccuracy of analog circuit implementation makes the performance of the system reduce. The system of the paper only needs to design baseband ADC with the same bandwidth, which reduces the design requirements of sampling hold circuit.
So the difficulty of overall system circuit implementation is less than that of HFB system in literature [15].

As the limited implementation accuracy of digital filter coefficient has an influence on reconstruction performance of analog-to-digital converter system for the real implementation of the system, Table 3 shows the length of eight-channel digitally synthesized filter, \( N = 128 \). The coefficient quantizes the reconstruction performance after 16-byte and 32-byte characteristic points.

We can see from Table 3 that after synthesized filter coefficient passes 16-bit quantization, the maximal overall aliasing error of the system is -59dB, and the effective number of the system still reach 9.5 bit [25]. In addition, under the same condition, the system reconstruction performance of the filter coefficient after 32-byte quantization has no reduction compared with that of the system with unlimited precision.

Table I. Performances of modulated ADC for two-channel case

<table>
<thead>
<tr>
<th></th>
<th>Average amplitude distortion /dB</th>
<th>Maximal amplitude distortion /dB</th>
<th>Average overall aliasing amplitude /dB</th>
<th>Maximal overall aliasing amplitude degree /dB</th>
<th>Average phase distortion /rad</th>
<th>Maximal phase distortion /rad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without sampling and guard band</td>
<td>(-6.4 \times 10^{-4})</td>
<td>(-0.25)</td>
<td>(-61)</td>
<td>(-24)</td>
<td>(-3.0 \times 10^{-7})</td>
<td>(1.0 \times 10^{-2})</td>
</tr>
<tr>
<td>Sampling 4% and guard band 3.5%</td>
<td>(-5.0 \times 10^{-4})</td>
<td>(2.8 \times 10^{-4})</td>
<td>(-112)</td>
<td>(-89)</td>
<td>(-2.4 \times 10^{-11})</td>
<td>(1.5 \times 10^{-9})</td>
</tr>
</tbody>
</table>

Table II. Performances of ADCs synthesized with different architectures

<table>
<thead>
<tr>
<th>Structure in literature [15]</th>
<th>Average amplitude distortion /dB</th>
<th>Maximal amplitude distortion /dB</th>
<th>Average overall aliasing amplitude /dB</th>
<th>Maximal overall aliasing amplitude degree /dB</th>
<th>Average phase distortion /rad</th>
<th>Maximal phase distortion /rad</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structure in the paper</td>
<td>(-3.1 \times 10^{-7})</td>
<td>(-2.1 \times 10^{-4})</td>
<td>(-97)</td>
<td>(-86)</td>
<td>(2.8 \times 10^{-7})</td>
<td>(1.2 \times 10^{-4})</td>
</tr>
</tbody>
</table>

Table III. Modulated ADC performances in the presence of coefficient quantization for eight-channel case

<table>
<thead>
<tr>
<th>Quantization</th>
<th>Average amplitude distortion /dB</th>
<th>Maximal amplitude distortion /dB</th>
<th>Average overall aliasing amplitude /dB</th>
<th>Maximal overall aliasing amplitude degree /dB</th>
<th>Average phase distortion /rad</th>
<th>Maximal phase distortion /rad</th>
</tr>
</thead>
<tbody>
<tr>
<td>16-bit</td>
<td>(3.0 \times 10^{-4})</td>
<td>(4.9 \times 10^{-4})</td>
<td>(-73)</td>
<td>(-59)</td>
<td>(7.7 \times 10^{-6})</td>
<td>(4.5 \times 10^{-4})</td>
</tr>
<tr>
<td>32-bit</td>
<td>(-2.2 \times 10^{-4})</td>
<td>(4.2 \times 10^{-4})</td>
<td>(-103)</td>
<td>(-79)</td>
<td>(3.1 \times 10^{-11})</td>
<td>(4.0 \times 10^{-4})</td>
</tr>
</tbody>
</table>

Figure 3. Distortion and total aliasing of modulated ADC for two-channel case

Figure 4. Total aliasing of modulated ADC for two-channel case with oversampling and guard band

IV. Conclusion

The paper proposes a new parallel modulation hybrid filter bank analog-digital conversion structure. The structure makes cosine modulation on broadband input signal, which is easy to use the same analog low-pass filter to segment signal frequency band, and the structure uses the narrow-band ADC sample with the same bandwidth to quantize sub-band signal. Each sub-band signal is converted into baseband for sampling quantization, and the structure proposed in the paper...
solves the problem that the other parallel ADC structures need high-speed sampling hold circuit, so the implementation of the structure proposed in the paper is not complicated as that of the other parallel ADC structures. The subsequent content is to study the influence of analog multiplier and ADC on system reconstruction performance under the condition of non-ideal implementation.

REFERENCES


